

SHARP

Worksheet 12 Memo – Analytical Geometry

Grade 10 – Mathematics

1. a) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$
 $M_E = \left(\frac{4+(-2)}{2}; \frac{4+(-2)}{2}\right)$
 $M_E = (1; 1)$

b) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$
 $(1; 1) = \left(\frac{x+2}{2}; \frac{y+0}{2}\right)$
 $\therefore 2 = x + 2 \text{ and } 2 = y$
 $x = 0$
B (0; 2)

c) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $d_{AB} = \sqrt{(0 - 4)^2 + (2 - 4)^2}$
 $\therefore d_{AB} = \sqrt{16 + 4}$
 $\therefore d_{AB} = \sqrt{20}$
 $\therefore d_{AB} = 2\sqrt{5} = 4.47$

$$d_{BC} = \sqrt{(0 - (-2))^2 + (2 - (-2))^2}$$
$$\therefore d_{BC} = \sqrt{4 + 16}$$
$$\therefore d_{BC} = \sqrt{20}$$
$$\therefore d_{BC} = 2\sqrt{5} = 4.47$$

$$d_{CD} = \sqrt{(-2 - 2)^2 + (-2 - 0)^2}$$
$$\therefore d_{CD} = \sqrt{16 + 4}$$
$$\therefore d_{CD} = \sqrt{20}$$
$$\therefore d_{CD} = 2\sqrt{5} = 4.47$$

$$d_{AD} = \sqrt{(4 - 2)^2 + (4 - 0)^2}$$
$$\therefore d_{AD} = \sqrt{4 + 16}$$
$$\therefore d_{AD} = \sqrt{20}$$
$$\therefore d_{AD} = 2\sqrt{5} = 4.47$$

d) This shape is a rhombus – all four sides are equal.

e) A (4; 4) and C (-2; -2)
 $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$
 $\therefore m = \frac{4 - (-2)}{4 - (-2)}$
 $\therefore m = 1$

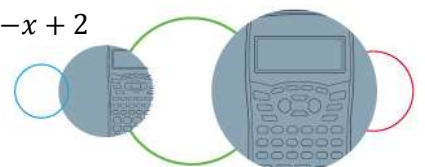
f) B (0; 2) and D (2; 0)
 $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$
 $\therefore m = \frac{2 - 0}{0 - 2}$
 $\therefore m = -1$

$$\therefore y = x + c$$
$$\therefore 4 = 4 + c$$
$$\therefore c = 0$$

$$\therefore y = -x + c$$
$$\therefore 2 = -(0) + c$$
$$\therefore c = 2$$

$$\therefore y = x$$

$$\therefore y = -x + 2$$



- g) $m_{AC} \times m_{BD} = 1 \times -1 = -1$
 $\therefore$ Line AC is perpendicular to line BD

2. a) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2} \right)$
 $\therefore M_E = \left(\frac{-5+3}{2}; \frac{0+4}{2} \right)$
 $\therefore M_E = (-1; 2)$

c) $\therefore m = \frac{y_2-y_1}{x_2-x_1}$
 $\therefore m_{AC} = \frac{5-2}{-2.5-(-1)}$
 $\therefore m_{AC} = -2$

$\therefore m_{AC} \times m_{BD} = -2 \times \frac{1}{2} = -1$
 $\therefore AEC \perp BED$

d) $d_{EB} = \sqrt{(-5 - (-1))^2 + (0 - 2)^2}$
 $\therefore d_{EB} = \sqrt{16 + 4}$
 $\therefore d_{EB} = \sqrt{20}$
 $\therefore d_{EB} = 2\sqrt{5} = 4.47$

e) $Area \Delta ABE = \frac{1}{2} b \times h$
 $= \frac{1}{2} \times 2\sqrt{5} \times \frac{3\sqrt{5}}{2}$
 $= 7\frac{1}{2} units^2$

g) $y = mx - 5$
 $\therefore 4 = m(3) - 5$
 $\therefore 9 = 3m$
 $\therefore m = 3$
 $\therefore y = 3x - 5 \rightarrow DC \dots 1$

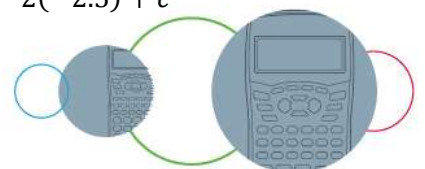
b) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\therefore d_{AB} = \sqrt{(-5 - (-2.5))^2 + (0 - 5)^2}$
 $\therefore d_{AB} = \sqrt{6\frac{1}{4} + 25}$
 $\therefore d_{AB} = \frac{5\sqrt{5}}{2} = 5.59$

$\therefore m_{BD} = \frac{0-4}{-5-3}$
 $\therefore m_{BD} = \frac{1}{2}$

$AB^2 = AE^2 + BE^2$
 $\therefore \left(\frac{5\sqrt{5}}{2} \right)^2 = AE^2 + (2\sqrt{5})^2$
 $\therefore AE^2 = 31\frac{1}{4} - 20$
 $\therefore AE^2 = 11\frac{1}{4}$
 $\therefore AE = \frac{3\sqrt{5}}{2} = 3.35$

f) $DC^2 = ED^2 + EC^2$
 $(2\sqrt{10})^2 = (2\sqrt{5})^2 + EC^2$
 $EC^2 = 40 - 20$
 $EC^2 = 20$
 $EC = \sqrt{20} = 2\sqrt{5} = 4.47$
 $\therefore AC = 3.35 + 4.47 = 7.82$

$AE \rightarrow y = -2x + c$
 $\therefore 5 = -2(-2.5) + c$



$$\therefore c = 0$$

Subs 2 into 1

$$\therefore y = -2x \quad \dots 2$$

$$-2x = 3x - 5$$

$$\therefore -5x = -5$$

$$\therefore x = 1 \quad \text{Subs back into 2:}$$

$$\therefore y = -2(1)$$

$$\therefore y = -2$$

$$\therefore C \text{ is } (1; -2)$$

$$h) \quad d_{AD} = \sqrt{(-2.5 - 3)^2 + (5 - 4)^2}$$

$$d_{BC} = \sqrt{(-5 - 1)^2 + (0 - (-2))^2}$$

$$\therefore d_{AD} = \sqrt{30\frac{1}{4} + 1}$$

$$\therefore d_{BC} = \sqrt{36 + 4}$$

$$\therefore d_{AD} = \sqrt{31\frac{1}{4}}$$

$$\therefore d_{BC} = \sqrt{40}$$

$$\therefore d_{AD} = \frac{5\sqrt{5}}{2} = 5.59$$

$$\therefore d_{BC} = 2\sqrt{10} = 6.32$$

i) ABCD is a kite. AD = AB and DC = BC.
Diagonals intersect.

$$3. \quad a) \quad M_C = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$\therefore M_C = \left(\frac{-2 + (-11)}{2}; \frac{2 + (-7)}{2} \right)$$

$$\therefore M_C = \left(-6\frac{1}{2}; -2\frac{1}{2} \right)$$

$$b) \quad \therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore y = -1x + c$$

$$\therefore m_{BD} = \frac{2 - (-7)}{-2 - (-11)}$$

$$\therefore -2\frac{1}{2} = -1\left(-6\frac{1}{2}\right) + c$$

$$\therefore m_{BD} = 1$$

$$\therefore c = -9$$

$$\therefore m_{AE} = -1 \div 1 = -1$$

$$\therefore y_{AE} = -x - 9$$

$$\therefore y = -(0) - 9$$

$$\therefore y = -9$$

$$\therefore \text{And } E(0; -9)$$

$$c) \quad y = -x - 9 \dots 1 \quad \text{and}$$

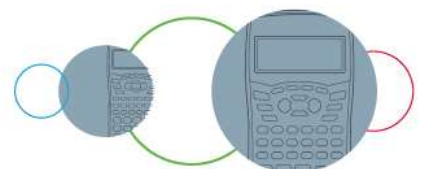
$$8y = x + 18 \dots 2$$

Subs 1 into 2:

$$\therefore 8(-x - 9) = x + 18$$

$$\therefore -8x - 72 = x + 18$$

$$\therefore -9x = 90$$



$$\therefore x = -10$$

Subs back into 1:

$$y = -(-10) - 9$$

$$\therefore y = 1$$

$$\therefore A (-10; 1)$$

d) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\therefore d_{AD} = \sqrt{(-10 - (-11))^2 + (1 - (-7))^2} \quad \therefore d_{AB} = \sqrt{(-10 - (-2))^2 + (1 - 2)^2}$$

$$\therefore d_{AD} = \sqrt{1 + 64}$$

$$\therefore d_{AB} = \sqrt{64 + 1}$$

$$\therefore d_{AD} = \sqrt{65}$$

$$\therefore d_{AB} = \sqrt{65}$$

$$\therefore d_{AD} = 8.06$$

$$\therefore d_{AB} = 8.06$$

e) The triangle is an isosceles triangle as two sides (AD and AB) are equal.

f) i) $d_{AB} = \sqrt{(-10 - (-2))^2 + (1 - 0)^2}$

$$\therefore d_{AB} = \sqrt{64 + 1}$$

$$\therefore d_{AB} = \sqrt{65} = 8.06$$

ii) ABD is still an equilateral triangle as AD is still equal to AB.

4. a) N (4; -5) and P (6; 5)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore m = \frac{5 - (-5)}{6 - 4}$$

$$\therefore m = 5$$

$$\therefore y = 5x + c$$

$$\therefore 5 = 5(6) + c$$

$$\therefore c = -25$$

$$\therefore y = 5x - 25$$

b) M (1; 6) and P(6; 5)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore m = \frac{6 - 5}{1 - 6}$$

$$\therefore m = -\frac{1}{5}$$

$$\therefore y = -\frac{1}{5}x + c$$

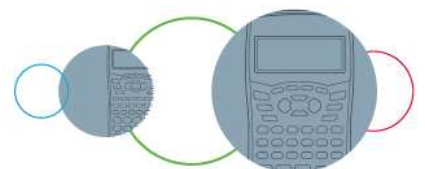
$$\therefore 5 = -\frac{1}{5}(6) + c$$

$$\therefore c = 6\frac{1}{5}$$

$$\therefore y = -\frac{1}{5}x + 6\frac{1}{5}$$

c) $m_{NP} \times m_{MP} = 5 \times -\frac{1}{5} = -1$

$\therefore$ Triangle MNP is a right angled triangle.



$$\begin{aligned}
 \text{d)} \quad M_Q &= \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2} \right) & M_R &= \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2} \right) & M_S &= \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2} \right) \\
 \therefore M_Q &= \left(\frac{1+4}{2}; \frac{6+(-5)}{2} \right) & \therefore M_R &= \left(\frac{4+6}{2}; \frac{-5+5}{2} \right) & \therefore M_S &= \left(\frac{1+6}{2}; \frac{6+5}{2} \right) \\
 \therefore M_Q &= \left(2\frac{1}{2}; \frac{1}{2} \right) & \therefore M_R &= (5; 0) & \therefore M_S &= \left(3\frac{1}{2}; 5\frac{1}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{e)} \quad \therefore m &= \frac{y_2-y_1}{x_2-x_1} & \therefore m &= \frac{y_2-y_1}{x_2-x_1} \\
 \therefore m_{QP} &= \frac{\frac{1}{2}-5}{2\frac{1}{2}-6} & \therefore m_{MR} &= \frac{0-6}{5-1} \\
 \therefore m_{QP} &= \frac{9}{7} & \therefore m_{MR} &= -\frac{3}{2} \\
 \therefore y &= \frac{9}{7}x + c & \therefore y &= -\frac{3}{2}x + c \\
 \therefore 5 &= \frac{9}{7}(6) + c & \therefore 6 &= -\frac{3}{2}(1) + c \\
 \therefore c &= -2\frac{5}{7} & \therefore c &= 7\frac{1}{2} \\
 \therefore y &= \frac{9}{7}x - 2\frac{5}{7} \dots 1 & \text{and} & y = -\frac{3}{2}x + 7\frac{1}{2} \dots 2 \\
 \text{Subs 1 into 2:} & & & \\
 \therefore \frac{9}{7}x - 2\frac{5}{7} &= -\frac{3}{2}x + 7\frac{1}{2} \\
 \therefore 2\frac{11}{14}x &= 10\frac{3}{14} \\
 \therefore x &= 3\frac{2}{3} & \text{Subs back into 2:} & y = -\frac{3}{2}\left(3\frac{1}{3}\right) + 7\frac{1}{2} \\
 & & & \therefore y = 2 \\
 T &= \left(3\frac{2}{3}; 2 \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{f)} \quad \text{Area } \triangle MNP &= \frac{1}{2} b \times h & \text{Area } \triangle NPR &= \frac{1}{2} b \times h \\
 &= \frac{1}{2} \times NP \times MP & &= \frac{1}{2} \times NP \times RP \\
 d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} & \therefore d_{MP} &= \sqrt{(1-6)^2 + (6-5)^2} \\
 \therefore d_{NP} &= \sqrt{(4-6)^2 + (-5-5)^2} & \therefore d_{MP} &= \sqrt{25+1} \\
 \therefore d_{NP} &= \sqrt{4+100} & \therefore d_{MP} &= \sqrt{26} \\
 \therefore d_{NP} &= \sqrt{104} = 2\sqrt{26} & & \\
 \text{And } RP &= \frac{1}{2} MP \\
 \therefore \text{Area } \triangle MNP &= \frac{1}{2} \times 2\sqrt{26} \times \sqrt{26} & \therefore \text{Area } \triangle NPR &= \frac{1}{2} \times 2\sqrt{26} \times \frac{1}{2}\sqrt{26} \\
 &= 26 \text{ units}^2 & &= 13 \text{ units}^2
 \end{aligned}$$

