Worksheet 12 Memo – Analytical Geometry

Grade 10 – Mathematics

1. a) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$M_E = \left(\frac{4+(-2)}{2}; \frac{4+(-2)}{2}\right)$

$M_E = (1; 1)$

b) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$(1; 1) = \left(\frac{x+2}{2}; \frac{y+0}{2}\right)$

$\therefore 2 = x + 2$ and $2 = y$

$x = 0$

B (0; 2)

c) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{AB} = \sqrt{(0-4)^2 + (2-4)^2}$

$\therefore d_{AB} = \sqrt{16+4}$

$\therefore d_{AB} = \sqrt{20}$

$\therefore d_{AB} = 2\sqrt{5} = 4.47$

$d_{BC} = \sqrt{\left(0-(-2)\right)^2 + \left(2-(-2)\right)^2}$

$\therefore d_{BC} = \sqrt{4+16}$

$\therefore d_{BC} = \sqrt{20}$

$\therefore d_{BC} = 2\sqrt{5} = 4.47$

$d_{CD} = \sqrt{(-2-2)^2 + (-2-0)^2}$

$\therefore d_{CD} = \sqrt{16+4}$

$\therefore d_{CD} = \sqrt{20}$

$\therefore d_{CD} = 2\sqrt{5} = 4.47$

$d_{AD} = \sqrt{(4-2)^2 + (4-0)^2}$

$\therefore d_{AD} = \sqrt{4+16}$

$\therefore d_{AD} = \sqrt{20}$

$\therefore d_{AD} = 2\sqrt{5} = 4.47$

d) This shape is a rhombus – all four sides are equal.

e) A (4; 4) and C (-2; -2)

$\therefore m = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m = \frac{4-(-2)}{4-(-2)}$

$\therefore m = 1$

$\therefore y = x + c$

$\therefore 4 = 4 + c$

$\therefore c = 0$

$\therefore y = x$

f) B (0; 2) and D (2; 0)

$\therefore m = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m = \frac{2-0}{0-2}$

$\therefore m = -1$

$\therefore y = -x + c$

$\therefore 2 = -(0) + c$

$\therefore c = 2$

$\therefore y = -x + 2$

g) $m_{AC} \times m_{BD} = 1 \times -1 = -1$

$\therefore$ Line AC is perpendicular to line BD

2. a) $M_E = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$\therefore M_E = \left(\frac{-5+3}{2}; \frac{0+4}{2}\right)$

$\therefore M_E = (-1; 2)$

b) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore d_{AB} = \sqrt{\left(-5 - (-2.5)\right)^2 + (0 - 5)^2}$

$\therefore d_{AB} = \sqrt{6\frac{1}{4} + 25}$

$\therefore d_{AB} = \frac{5\sqrt{5}}{2} = 5.59$

c) $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{5-2}{-2.5-(-1)}$

$\therefore m_{AC} = -2$

$\therefore m_{BD} = \frac{0-4}{-5-3}$

$\therefore m_{BD} = \frac{1}{2}$

$\therefore m_{AC} \times m_{BD} = -2 \times \frac{1}{2} = -1$

$\therefore AEC \perp BED$

d) $d_{EB} = \sqrt{\left(-5 - (-1)\right)^2 + (0 - 2)^2}$

$\therefore d_{EB} = \sqrt{16 + 4}$

$\therefore d_{EB} = \sqrt{20}$

$\therefore d_{EB} = 2\sqrt{5} = 4.47$

$AB^2 = AE^2 + BE^2$

$\therefore \left(\frac{5\sqrt{5}}{2}\right)^2 = AE^2 + \left(2\sqrt{5}\right)^2$

$\therefore AE^2 = 31\frac{1}{4} - 20$

$\therefore AE^2 = 11\frac{1}{4}$

$\therefore AE = \frac{3\sqrt{5}}{2} = 3.35$

e) $Area\ \Delta ABE = \frac{1}{2}b \times h$

$= \frac{1}{2} \times 2\sqrt{5} \times \frac{3\sqrt{5}}{2}$

$= 7\frac{1}{2}\ units^2$

f) $DC^2 = ED^2 + EC^2$

$\left(2\sqrt{10}\right)^2 = \left(2\sqrt{5}\right)^2 + EC^2$

$EC^2 = 40 - 20$

$EC^2 = 20$

$EC = \sqrt{20} = 2\sqrt{5} = 4.47$

$\therefore AC = 3.35 + 4.47 = 7.82$

g) $y = mx - 5$

$\therefore 4 = m(3) - 5$

$\therefore 9 = 3m$

$\therefore m = 3$

$\therefore y = 3x - 5 \quad \rightarrow DC \quad \ldots.1$

$AE \rightarrow y = -2x + c$

$\therefore 5 = -2(-2.5) + c$

$\therefore c = 0$

$\therefore y = -2x$ …2

Subs 2 into 1

$-2x = 3x - 5$

$\therefore -5x = -5$

$\therefore x = 1$ Subs back into 2: $\therefore y = -2(1)$

$\therefore y = -2$

∴ C is (1; -2)

h) $d_{AD} = \sqrt{(-2.5-3)^2 + (5-4)^2}$

$\therefore d_{AD} = \sqrt{30\frac{1}{4} + 1}$

$\therefore d_{AD} = \sqrt{31\frac{1}{4}}$

$\therefore d_{AD} = \frac{5\sqrt{5}}{2} = 5.59$

$d_{BC} = \sqrt{(-5-1)^2 + \left(0-(-2)\right)^2}$

$\therefore d_{BC} = \sqrt{36+4}$

$\therefore d_{BC} = \sqrt{40}$

$\therefore d_{BC} = 2\sqrt{10} = 6.32$

i) ABCD is a kite. AD = AB and DC = BC.

Diagonals intersect.

3. a) $M_C = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$\therefore M_C = \left(\frac{-2+(-11)}{2}; \frac{2+(-7)}{2}\right)$

$\therefore M_C = \left(-6\frac{1}{2}; -2\frac{1}{2}\right)$

b) $\therefore m = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{BD} = \frac{2-(-7)}{-2-(-11)}$

$\therefore m_{BD} = 1$

$\therefore y = -1x + c$

$\therefore -2\frac{1}{2} = -1\left(-6\frac{1}{2}\right) + c$

$\therefore c = -9$

$\therefore m_{AE} = -1 \div 1 = -1$

$\therefore y_{AE} = -x - 9$

$\therefore y = -(0) - 9$

$\therefore y = -9$

∴And E (0; -9)

c) $y = -x - 9$ ….1 and $8y = x + 18$ …2

Subs 1 into 2:

$\therefore 8(-x-9) = x + 18$

$\therefore -8x - 72 = x + 18$

$\therefore -9x = 90$

$\therefore x = -10$ Subs back into 1: $y = -(-10) - 9$

$\therefore y = 1$

$\therefore$ A (-10; 1)

d) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore d_{AD} = \sqrt{\left(-10 - (-11)\right)^2 + \left(1 - (-7)\right)^2}$

$\therefore d_{AD} = \sqrt{1 + 64}$

$\therefore d_{AD} = \sqrt{65}$

$\therefore d_{AD} = 8.06$

$\therefore d_{AB} = \sqrt{\left(-10 - (-2)\right)^2 + (1 - 2)^2}$

$\therefore d_{AB} = \sqrt{64 + 1}$

$\therefore d_{AB} = \sqrt{65}$

$\therefore d_{AB} = 8.06$

e) The triangle is an isosceles triangle as two sides (AD and AB) are equal.

f) i) $d_{AB} = \sqrt{\left(-10 - (-2)\right)^2 + (1 - 0)^2}$

$\therefore d_{AB} = \sqrt{64 + 1}$

$\therefore d_{AB} = \sqrt{65} = 8.06$

ii) ABD is still an equilateral triangle as AD is still equal to AB.

4. a) N (4; -5) and P (6; 5)

$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m = \frac{5-(-5)}{6-4}$

$\therefore m = 5$

$\therefore y = 5x + c$

$\therefore 5 = 5(6) + c$

$\therefore c = -25$

$\therefore y = 5x - 25$

b) M (1; 6) and P(6; 5)

$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m = \frac{6-5}{1-6}$

$\therefore m = -\frac{1}{5}$

$\therefore y = -\frac{1}{5}x + c$

$\therefore 5 = -\frac{1}{5}(6) + c$

$\therefore c = 6\frac{1}{5}$

$\therefore y = -\frac{1}{5}x + 6\frac{1}{5}$

c) $m_{NP} \times m_{MP} = 5 \times -\frac{1}{5} = -1$

$\therefore$ Triangle MNP is a right angled triangle.

d) $M_Q = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $\quad M_R = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $\quad M_S = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$\therefore M_Q = \left(\frac{1+4}{2};\frac{6+(-5)}{2}\right)$ $\quad \therefore M_R = \left(\frac{4+6}{2};\frac{-5+5}{2}\right)$ $\quad \therefore M_S = \left(\frac{1+6}{2};\frac{6+5}{2}\right)$

$\therefore M_Q = \left(2\frac{1}{2};\frac{1}{2}\right)$ $\quad \therefore M_R = (5;0)$ $\quad \therefore M_S = \left(3\frac{1}{2};5\frac{1}{2}\right)$

e) $\therefore m = \frac{y_2-y_1}{x_2-x_1}$ $\quad \therefore m = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{QP} = \frac{\frac{1}{2}-5}{2\frac{1}{2}-6}$ $\quad \therefore m_{MR} = \frac{0-6}{5-1}$

$\therefore m_{QP} = \frac{9}{7}$ $\quad \therefore m_{MR} = -\frac{3}{2}$

$\therefore y = \frac{9}{7}x + c$ $\quad \therefore y = -\frac{3}{2}x + c$

$\therefore 5 = \frac{9}{7}(6) + c$ $\quad \therefore 6 = -\frac{3}{2}(1) + c$

$\therefore c = -2\frac{5}{7}$ $\quad \therefore c = 7\frac{1}{2}$

$\therefore y = \frac{9}{7}x - 2\frac{5}{7}$ …1 and $y = -\frac{3}{2}x + 7\frac{1}{2}$ … 2

Subs 1 into 2:

$\therefore \frac{9}{7}x - 2\frac{5}{7} = -\frac{3}{2}x + 7\frac{1}{2}$

$\therefore 2\frac{11}{14}x = 10\frac{3}{14}$

$\therefore x = 3\frac{2}{3}$ Subs back into 2: $y = -\frac{3}{2}\left(\frac{11}{3}\right) + 7\frac{1}{2}$

$\therefore y = 2$

$T\left(3\frac{2}{3};2\right)$

f) $Area\ \Delta MNP = \frac{1}{2}b \times h$ $\quad Area\ \Delta NPR = \frac{1}{2}b \times h$

$= \frac{1}{2} \times NP \times MP$ $\quad = \frac{1}{2} \times NP \times RP$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\therefore d_{NP} = \sqrt{(4-6)^2 + (-5-5)^2}$ $\quad \therefore d_{MP} = \sqrt{(1-6)^2 + (6-5)^2}$

$\therefore d_{NP} = \sqrt{4+100}$ $\quad \therefore d_{MP} = \sqrt{25+1}$

$\therefore d_{NP} = \sqrt{104} = 2\sqrt{26}$ $\quad \therefore d_{MP} = \sqrt{26}$

And $RP = \frac{1}{2}MP$

$\therefore Area\ \Delta\ MNP = \frac{1}{2} \times 2\sqrt{26} \times \sqrt{26}$ $\quad \therefore Area\ \Delta\ NPR = \frac{1}{2} \times 2\sqrt{26} \times \frac{1}{2}\sqrt{26}$

$= 26\ units^2$ $\quad = 13\ units^2$