SHARP

# Worksheet 4 Memo – Exponents

## Grade 10 Mathematics

1.

| Power | Base | Exponent | Value |
|---|---|---|---|
| $2^3$ | 2 | 3 | 8 |
| $4^2$ | 4 | 2 | 16 |
| $5^6$ | 5 | 6 | 15 625 |
| $3^0$ | 3 | 0 | 1 |
| $2^{-1}$ | 2 | -1 | $\frac{1}{2}$ |

2. i) When the same bases are multiplied the exponents are added

$a^n \times a^m = a^{n+m}$

ii) When the same bases are divided the exponents are subtracted

$a^n \div a^m = a^{n-m}$

iii) When a power is raised to an exponent, the exponents are multiplied

$(a^n)^m = a^{n\times m}$

iv) When a power is under a root sign the exponent is divided by the root

$\sqrt[m]{a^n} = a^{\frac{n}{m}}$

v) Anything to the power of zero is one → $a^0 = 1$

vi) When a power has a negative exponent, the power is inverted (placed under 1) and the exponent becomes positive. $a^{-n} = \frac{1}{a^n}$

3. a) $a^{2x}.a^{3x+1}$

$= a^{5x+1}$

b) $2^x.8^{x+1}$

$= 2^x.2^{3(x+1)}$

$= 2^x.2^{3x+3}$

$= 2^{4x+3}$

c) $\frac{3^{2x}.27^{x-1}}{9^{2x+2}}$

$= \frac{3^{2x}.3^{3(x-1)}}{3^{2(2x+2)}}$

$= \frac{3^{2x}.3^{3x-3}}{3^{4x+4}}$

$= 3^{2x+3x-3-(4x+4)}$

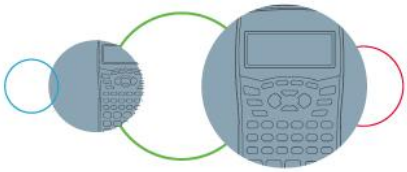
$= 3^{5x-3-4x-4}$

d) $\frac{a^2b^3 \times a^3b^4}{(a^3b^3)^2}$

$= \frac{a^5b^7}{a^6b^6}$

$= \frac{b}{a}$





$= 3^{x-7}$

e) $\frac{(ab^2c)^3}{a^{-1}b^4c^3} \times \frac{(a^{-2}b^3c^5)^0}{a^{-1}b^4c^2}$

$= \frac{a^3b^6c^3}{a^{-1}b^4c^3} \times \frac{1}{a^{-1}b^4c^2}$

$= \frac{a^3b^6c^3}{a^{-2}b^8c^5}$

$= \frac{a^5}{b^2c^2}$

f) $\frac{x^ay^{2a}}{x^{2a-1}} \div \frac{(x^2y)^a}{x^{3+a}y^{-1}}$

$= \frac{x^ay^{2a}}{x^{2a-1}} \times \frac{x^{3+a}y^{-1}}{x^{2a}y^{-a}}$

$= x^{a+3+a-(2a-1+2a)}.y^{2a-1-(-a)}$

$= x^{2a+3-4a+1}y^{2a-1+a}$

$= x^{-2a+4}y^{3a-1}$

g) $\frac{18^a.15^{a+1}}{30^{a-1}} \times \frac{(3^a.6)^2}{9^{a-1}} \div \frac{36^{a+1}}{18^{a-1}}$

$= \frac{(3^22)^a(3.5)^{a+1}}{(3.2.5)^{a-1}} \times \frac{3^{2a}.(3.2)^2}{3^{2(a-1)}} \times \frac{(3^2.2)^{a-1}}{(3^2.2^2)^{a+1}}$

$= \frac{3^{2a}2^a.3^{a+1}5^{a+1}}{3^{a-1}2^{a-1}5^{a-1}} \times \frac{3^{2a}.3^22^2}{3^{2a-2}} \times \frac{3^{2a-2}2^{a-1}}{3^{2a+2}.2^{2a+2}}$

$= 3^{2a+a+1-(a-1)}.2^{a+2-(2a+2)}.5^{a+1-(a-1)}$

$= 3^{3a+1-a+1}.2^{a+2-2a-2}.5^{a+1-a+1}$

$= 3^{2a+2}2^{-a}5^2$

$= \frac{3^{2a+2}5^2}{2^a}$

h) $\frac{(ab^2)^{-6} \times (a^3b^7)^2}{(a^0b^2)^{-\frac{1}{2}}}$

$= \frac{a^{-6}b^{-12} \times a^6b^{14}}{a^0b^{-1}}$

$= a^{-6+6+0}b^{-12+14-(-1)}$

$= a^0b^{2+1}$

$= b^3$

i) $\frac{3^{a-1}.6^{(a+2)}.9^{-a}}{18^{a-1}3^a}$

$= \frac{3^{a-1}.(3.2)^{a+2}.3^{2(-a)}}{(3^2.2)^{a-1}.3^a}$

$= \frac{3^{a-1}\,3^{a+2}2^{a+2}.3^{-2a}}{3^{2a-2}2^{a-1}.3^a}$

$= 3^{a-1+a+2-2a-(2a-2+a)}.2^{a+2-(a-1)}$

$= 3^{1-3a+2}.2^{a+2-a+1}$

$= 3^{3-3a}2^3$

j) $\frac{(xyz^2)^{-1}}{x^2y^{-1}z^3} \div \left(\frac{x^3y^2z^0}{x^4y^2z^{-3}}\right)^{-1} \times \frac{xy^2z^4}{x^{-3}y^2z^0}$

$= \frac{x^{-1}y^{-1}z^{-2}}{x^2y^{-1}z^3} \div \frac{x^4y^2z^{-3}}{x^3y^2} \times \frac{xy^2z^4}{x^{-3}y^2}$

$= \frac{x^{-1}z^{-2}}{x^2z^3} \times \frac{x^3}{x^4z^{-3}} \times \frac{xz^4}{x^{-3}}$

$= x^{-1+3+1-(2+4-3)}.z^{-2+4-(3-3)}$

$= x^{3-3}.z^2$

$= z^2$

4. a) $\frac{3^{x+1}-2.3^x}{3^x.2}$

$= \frac{3^x.3-2.3^x}{3^x.2}$

$= \frac{3^x(3-2)}{3^x.2}$

$= \frac{1}{2}$

b) $\frac{2^{2x+3}-5.2^{2x+1}}{4^{x+2}}$

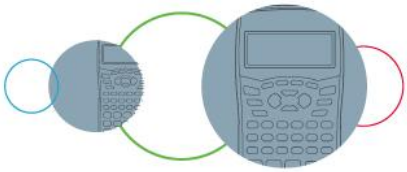
$= \frac{2^{2x}.2^3-5.2^{2x}.2}{2^{2(x+2)}}$

$= \frac{2^{2x+1}(2^2-5)}{2^{2x+4}}$

$= \frac{2^{2x+1}(4-5)}{2^{2x+4}}$

$= \frac{-1}{2^3} = -\frac{1}{8}$





c) $\frac{35^a-3.5^a}{2^{2a}7^a-3.2^{2a}}$

$=\frac{(7.5)^a-3.5^a}{2^{2a}(7^a-3)}$

$=\frac{7^a5^a-3.5^a}{2^{2a}(7^a-3)}$

$=\frac{5^a(7^a-3)}{2^{2a}(7^a-3)}$

$=\frac{5^a}{2^{2a}}$

$=\left(\frac{5}{2^2}\right)^a$

$=\left(\frac{5}{4}\right)^a$

d) $\frac{3^{a+1}.4^a+5.3^{a+1}}{4^{2a}-25}$

$=\frac{3^{a+1}(4^a+5)}{(4^a-5)(4^a+5)}$

$=\frac{3^{a+1}}{4^a-5}$

e) $\frac{7^a.49-7^{a+2}.2^{-1}}{2^{-3}.7^a}$

$=\frac{7^a.7^2-7^{a+2}.2^{-1}}{2^{-3}.7^a}$

$=\frac{7^{a+2}\left(1-\frac{1}{2}\right)}{7^a(2^{-3})}$

$=\frac{49\left(\frac{1}{2}\right)}{\frac{1}{8}}$

$=196$

f) $\frac{2^{3a-1}+\frac{3}{2}}{2^{4a-1}+3.2^{a-1}}$

$=\frac{2^{3a}.2^{-1}+3.2^{-1}}{2^a2^{3a}.2^{-1}+3.2^a.2^{-1}}$

$=\frac{2^{-1}\left(2^{3a}+3\right)}{2^{a-1}(2^{3a}+3)}$

$=2^{-1-(a-1)}$

$=2^{-1-a+1}$

$=2^{-a}$

$=\frac{1}{2^a}$

g) $\frac{-2^3-2^5}{5^{a+1}.2^2}$

$=\frac{-2^3\left(1+2^2\right)}{2^2(5^{a+1})}$

$=\frac{-2(5)}{5^a.5}$

$=-\frac{2}{5^a}$

h) $\frac{\frac{3}{4^2}+4^{a-2}}{3.4^3+4^{a+3}}$

$=\frac{3.4^{-2}+4^a.4^{-2}}{3.4^3+4^a.4^3}$

$=\frac{4^{-2}(3+4^a)}{4^3(3+4^a)}$

$=\frac{1}{4^5}$

i) $\frac{3^{a+1}+3^{a+3}}{3^{a+4}-3^{a+2}}$

$=\frac{3^{a+1}\left(1+3^2\right)}{3^{a+1}(3^3-3)}$

$=\frac{1+9}{27-3}$

$=\frac{10}{24}$

$=\frac{5}{12}$

j) $\frac{5.2^{x-1}-\frac{3.2^x}{2}}{3.2^x-2^x}$

$=\frac{5.2^x.2^{-1}-3.2^x.2^{-1}}{3.2^x-2^x}$

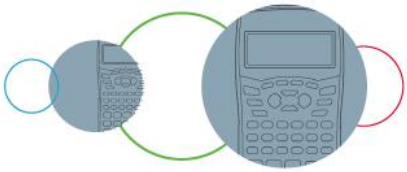
$=\frac{2^{x-1}(5-3)}{2^x(3-1)}$

$=\frac{2^{-1}(2)}{(2)}$

$=2^{-1}$

$=\frac{1}{2}$





5. a) $4.3^{3x+2} + 3^{3x+3} = 7$

$3^{3x+2}(4 + 3^1) = 7$

$3^{3x+2}(7) = 7$

$3^{3x+2} = 1 = 3^0$

$\therefore 3x + 2 = 0$

$\therefore x = -\frac{2}{3}$

b) $2^x - 2^{x-2} = 6$

$2^x(1 - 2^{-2}) = 6$

$2^x\left(\frac{3}{4}\right) = 6$

$2^x = 8 = 2^3$

$\therefore x = 3$

c) $7.3^x = 217$

$3^x = 31$

$x$ is between 3 and 4

$x = 3.12$

d) $3.5^{x+1} = 375$

$5^{x+1} = 125$

$5^{x+1} = 5^3$

$x + 1 = 3$

$x = 2$

e) $5^{x-1} - 3.5^{x+1} + 10\,000 = 750$

$5^{x-1} - 3.5^{x+1} = -9\,250$

$5^x(5^{-1} - 3.5) = -9\,250$

$5^x\left(-\frac{74}{5}\right) = -9\,250$

$5^x = 625$

$5^x = 5^4$

$x = 4$

f) $3^2.6^{x-1} - 2^3.6^{x-1} = \frac{1}{6}$

$6^{x-1}(3^2 - 2^3) = \frac{1}{6}$

$6^{x-1}(9 - 8) = \frac{1}{6}$

$6^{x-1} = 6^{-1}$

$x - 1 = -1$

$x = 0$

g) $7^{2x+4} - 5.49^{x+1\frac{1}{2}} = 14$

$7^{2x}7^4 - 5.7^{2\left(x+1\frac{1}{2}\right)} = 14$

$7^{2x}7^4 - 5.7^{2x+3} = 14$

$7^{2x+3}(7 - 5) = 14$

$7^{2x+3} = 7^1$

$2x + 3 = 1$

$2x = -2$

$x = -1$

h) $8.6^{x-1} = 3\,912$ (to two decimal places)

$6^{x-1} = 489$

$x - 1$ lies between 3 and 4

$x - 1 = 3{,}45$

$x = 4{,}45$

i) $5.2^{2x+1} = 320$

$2^{2x+1} = 64$

$2^{2x+1} = 2^6$

$2x + 1 = 6$

$2x = 5$

$x = \frac{5}{2}$ *or* $2\frac{1}{2}$

j) $2^3 4^{x+2} - 4^{x-1}.8 = \frac{63}{32}$

$8.4^{x+2} - 8.4^{x-1} = \frac{63}{32}$

$8.4^x(4^2 - 4^{-1}) = \frac{63}{32}$

$8.4^x\left(\frac{63}{4}\right) = \frac{63}{32}$

$8.4^x = \frac{1}{8}$

$4^x = \frac{1}{64}$

$4^x = 4^{-3}$

$x = -3$