

SHARP

Worksheet 3 Memo – Algebra: Algebraic Fractions

Grade 10 Mathematics

1. a)
$$\frac{x^2 - y^2}{2(x+y)}$$
$$= \frac{(x-y)(x+y)}{2(x+y)}$$
$$= \frac{x-y}{2}$$

b)
$$\frac{-2x+4}{x-2}$$
$$= \frac{-2(x-2)}{(x-2)}$$
$$= -2$$

c)
$$\frac{x^2 - 3x - 10}{3x - 15}$$
$$= \frac{(x-5)(x+2)}{3(x-5)}$$
$$= \frac{x+2}{3}$$

d)
$$\frac{x^2 + 3x - 28}{x^2 - 16}$$
$$= \frac{(x+7)(x-4)}{(x-4)(x+4)}$$
$$= \frac{x+7}{x+4}$$

e)
$$\frac{(x+2)(x^2 - 36)}{x^2 + 8x + 12}$$
$$= \frac{(x+2)(x+6)(x-6)}{(x+6)(x+2)}$$
$$= x - 6$$

f)
$$\frac{2x^2 - 4x - 6}{2x^2 - 18}$$
$$= \frac{2(x^2 - 2x - 3)}{2(x^2 - 9)}$$
$$= \frac{(x-3)(x+1)}{(x-3)(x+3)}$$
$$= \frac{x+1}{x+3}$$

g)
$$\frac{8x^3 + 64y^3}{x^2 - xy - 6y^2}$$
$$= \frac{8(x^3 + 8y^3)}{(x-3y)(x+2y)}$$
$$= \frac{8(x+2y)(x^2 - 2xy + 4y^2)}{(x-3y)(x+2y)}$$
$$= \frac{8(x^2 - 2xy + 4y^2)}{x-3y}$$

h)
$$\frac{ac + bd + bc + ad}{c^2 - 2cd - 3d^2}$$
$$= \frac{ac + bc + bd + ad}{(c-3d)(c+d)}$$
$$= \frac{c(a+b) + d(b+a)}{(c-3d)(c+d)}$$
$$= \frac{(a+b)(c+d)}{(c-3d)(c+d)}$$
$$= \frac{a+b}{c-3d}$$

i)
$$\frac{x^2 - 9}{x^3 + 27}$$
$$= \frac{(x-3)(x+3)}{(x+3)(x^2 - 3x + 9)}$$
$$= \frac{x-3}{x^2 - 3x + 9}$$

j)
$$\frac{3a(a+b) - 4b(a+b)}{a^2 - b^2}$$
$$= \frac{(a+b)(3a-4b)}{(a-b)(a+b)}$$
$$= \frac{3a-4b}{a-b}$$

$$\begin{aligned}
 2. \quad a) \quad & \frac{x^2-y^2}{3} \times \frac{6x-3y}{x^3+y^3} \\
 &= \frac{(x-y)(x+y)}{3} \times \frac{3(2x-y)}{(x+y)(x^2-xy+y^2)} \\
 &= \frac{(x-y)(2x-y)}{x^2-xy+y^2} \\
 &= \frac{2x^2-3xy+y^2}{x^2-xy+y^2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{2x^3-16y^3}{4x+8y} \times \frac{x^2-4y^2}{x^2+2xy+4y^2} \\
 &= \frac{2(x^3-8y^3)}{4(x+2y)} \times \frac{(x-2y)(x+2y)}{x^2+2xy+4y^2} \\
 &= \frac{(x-2y)(x^2+2xy+4y^2)}{2(x+2y)} \times \frac{(x-2y)(x+2y)}{x^2+2xy+4y^2} \\
 &= \frac{(x-2y)(x-2y)}{2} \\
 &= \frac{x^2-4xy+4y^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{x^2+5x+6}{2x+4} \times \frac{x-3}{x^2-9} \\
 &= \frac{(x+3)(x+2)}{2(x+2)} \times \frac{x-3}{(x-3)(x+3)} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{x^2+3x-4}{x^3+64y^3} \div \frac{x^2-1}{x^2-4xy+16y^2} \\
 &= \frac{(x+4)(x-1)}{(x+4y)(x^2-4xy+16y^2)} \times \frac{x^2-4xy+16y^2}{(x-1)(x+1)} \\
 &= \frac{x+4}{(x+4y)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{x^2+4x-45}{x^2-25} \div \frac{x^2-5x+25}{x^3+125} \\
 &= \frac{(x+9)(x-5)}{(x-5)(x+5)} \times \frac{(x+5)(x^2-5x+25)}{x^2-5x+25} \\
 &= x+9
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{ax+ay}{x^2-y^2} \times \frac{x^2+2xy-3y^2}{a} \\
 &= \frac{a(x+y)}{(x-y)(x+y)} \times \frac{(x+3y)(x-y)}{a} \\
 &= x+3y
 \end{aligned}$$

$$\begin{aligned}
 g) \quad & \frac{x^2-16}{x^3+64} \div \frac{3x-12}{x^2-4x+16} \\
 &= \frac{(x-4)(x+4)}{(x+4)(x^2-4x+16)} \times \frac{x^2-4x+16}{3(x-4)} \\
 &= \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad & \frac{x^2-6x+5}{x^2+x+1} \div \frac{3x-15}{x^3-1} \\
 &= \frac{(x-5)(x-1)}{x^2+x+1} \times \frac{(x-1)(x^2+x+1)}{3(x-5)} \\
 &= \frac{x^2-1}{3}
 \end{aligned}$$

$$\begin{aligned}
 i) \quad & \frac{8x^3-27}{4x^2-9} \times \frac{8x+12}{4x^2+6x+9} \\
 &= \frac{(2x-3)(4x^2+6x+9)}{(2x-3)(2x+3)} \times \frac{4(2x+3)}{4x^2+6x+9} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 j) \quad & \frac{x^2-5x+6}{x^3-8} \div \frac{x^2-9}{x^3+2x^2+4x} \\
 &= \frac{(x-3)(x-2)}{(x-2)(x^2+2x+4)} \times \frac{x(x^2+2x+4)}{(x-3)(x+3)} \\
 &= \frac{x}{x+3}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad a) \quad & \frac{x+3}{x} + \frac{x+3}{4} \\
 &= \frac{4(x+3)+x(x+3)}{4x} \\
 &= \frac{4x+12+x^2+3x}{4x} \\
 &= \frac{x^2+7x+12}{4x}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{x}{x+1} - \frac{6}{x-1} \\
 &= \frac{x(x-1)-6(x+1)}{(x+1)(x-1)} \\
 &= \frac{x^2-x-6x-6}{x^2-1} \\
 &= \frac{x^2-7x-6}{x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & \frac{3}{x} + \frac{x+8}{x-4} - \frac{x^2+3}{x^2-4x} \\
 &= \frac{3}{x} + \frac{x+8}{x-4} - \frac{x^2+3}{x(x-4)} \\
 &= \frac{3(x-4) + x(x+8) - (x^2+3)}{x(x-4)} \\
 &= \frac{3x-12+x^2+8x-x^2-3}{x(x-4)} \\
 &= \frac{11x-15}{x^2-4x}
 \end{aligned}$$

$$\begin{aligned}
 \text{d)} \quad & \frac{x+2}{x^2-1} + \frac{x+5}{x-1} - \frac{x-3}{x+1} \\
 &= \frac{x+2}{(x-1)(x+1)} + \frac{x+5}{x-1} - \frac{x-3}{x+1} \\
 &= \frac{x+2+(x+5)(x+1)-(x-3)(x-1)}{(x-1)(x+1)} \\
 &= \frac{x+2+x^2+6x+5-(x^2-4x+3)}{(x-1)(x+1)} \\
 &= \frac{x^2+7x+7-x^2+4x-3}{(x-1)(x+1)} \\
 &= \frac{11x+4}{x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{e)} \quad & \frac{x-2}{x} + \frac{1}{3} + \frac{-2x}{x-4} \\
 &= \frac{3(x-2)(x-4)+1(x)(x-4)-2x(3x)}{3x(x-4)} \\
 &= \frac{3(x^2-6x+8)+x^2-4x-6x^2}{3x(x-4)} \\
 &= \frac{3x^2-18x+24+x^2-4x-6x^2}{3x(x-4)} \\
 &= \frac{-2x^2-22x+24}{3x^2+12x}
 \end{aligned}$$

$$\begin{aligned}
 \text{f)} \quad & \frac{3x-1}{x+2} - \frac{x-5}{3x} - \frac{4}{4x+8} \\
 &= \frac{3x-1}{x+2} - \frac{x-5}{3x} - \frac{4}{4(x+2)} \\
 &= \frac{(3x-1)(3x)-(x-5)(x+2)-1(3x)}{3x(x+2)} \\
 &= \frac{9x^2-3x-(x^2-3x-10)-3x}{3x(x+2)} \\
 &= \frac{9x^2-6x-x^2+3x+10}{3x(x+2)} \\
 &= \frac{8x^2-3x+10}{3x^2+6x}
 \end{aligned}$$

$$\begin{aligned}
 \text{g)} \quad & \frac{-6}{x+2} - \frac{-x+3}{x} + \frac{x-1}{3x} \\
 &= \frac{-6}{x+2} + \frac{x-3}{x} + \frac{x-1}{3x} \\
 &= \frac{-6(3x)+3(x-3)(x+2)+(x-1)(x+2)}{3x(x+2)} \\
 &= \frac{-18x+3(x^2-x-6)+x^2+x-2}{3x(x+2)} \\
 &= \frac{x^2-17x-2+3x^2-3x-18}{3x(x+2)} \\
 &= \frac{4x^2-20x-20}{3x^2+6x}
 \end{aligned}$$

$$\begin{aligned}
 \text{h)} \quad & \frac{x^2+7x}{x^2-4} + \frac{x-3}{2-x} - \frac{5}{x+2} \\
 &= \frac{x^2+7x}{(x-2)(x+2)} - \frac{x-3}{x-2} - \frac{5}{x+2} \\
 &= \frac{x^2+7x-(x-3)(x+2)-5(x-2)}{(x-2)(x+2)} \\
 &= \frac{x^2+7x-(x^2-x-6)-5x+10}{(x-2)(x+2)} \\
 &= \frac{x^2+2x+10-x^2+x+6}{(x-2)(x+2)} \\
 &= \frac{3x+16}{x^2-4}
 \end{aligned}$$

$$\begin{aligned}
 \text{i)} \quad & \frac{x}{x-1} + \frac{x+3}{x+1} - \frac{4}{1-x^2} \\
 &= \frac{x}{x-1} + \frac{x+3}{x+1} + \frac{4}{x^2-1} \\
 &= \frac{x}{x-1} + \frac{x+3}{x+1} + \frac{4}{(x-1)(x+1)} \\
 &= \frac{x(x+1)+(x+3)(x-1)+4}{(x-1)(x+1)} \\
 &= \frac{x^2+x+x^2+2x-3+4}{(x-1)(x+1)} \\
 &= \frac{2x^2+3x+1}{(x-1)(x+1)} = \frac{(x+1)(2x+1)}{(x-1)(x+1)} \\
 &= \frac{2x+1}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \frac{a}{ax+a} - \frac{x-2}{x+1} + \frac{x+2}{x-1} \\
 &= \frac{a}{a(x+1)} - \frac{x-2}{x+1} + \frac{x+2}{x-1} \\
 &= \frac{1(x-1)-(x-2)(x-1)+(x+2)(x+1)}{(x+1)(x-1)} \\
 &= \frac{x-1-(x^2-3x+2)+x^2+3x+2}{(x+1)(x-1)} \\
 &= \frac{x^2+4x+1-x^2+3x-2}{(x+1)(x-1)} = \frac{7x-1}{x^2-1}
 \end{aligned}$$