Grade 10

SHARP

Mathematics

Study Guide

Algebra

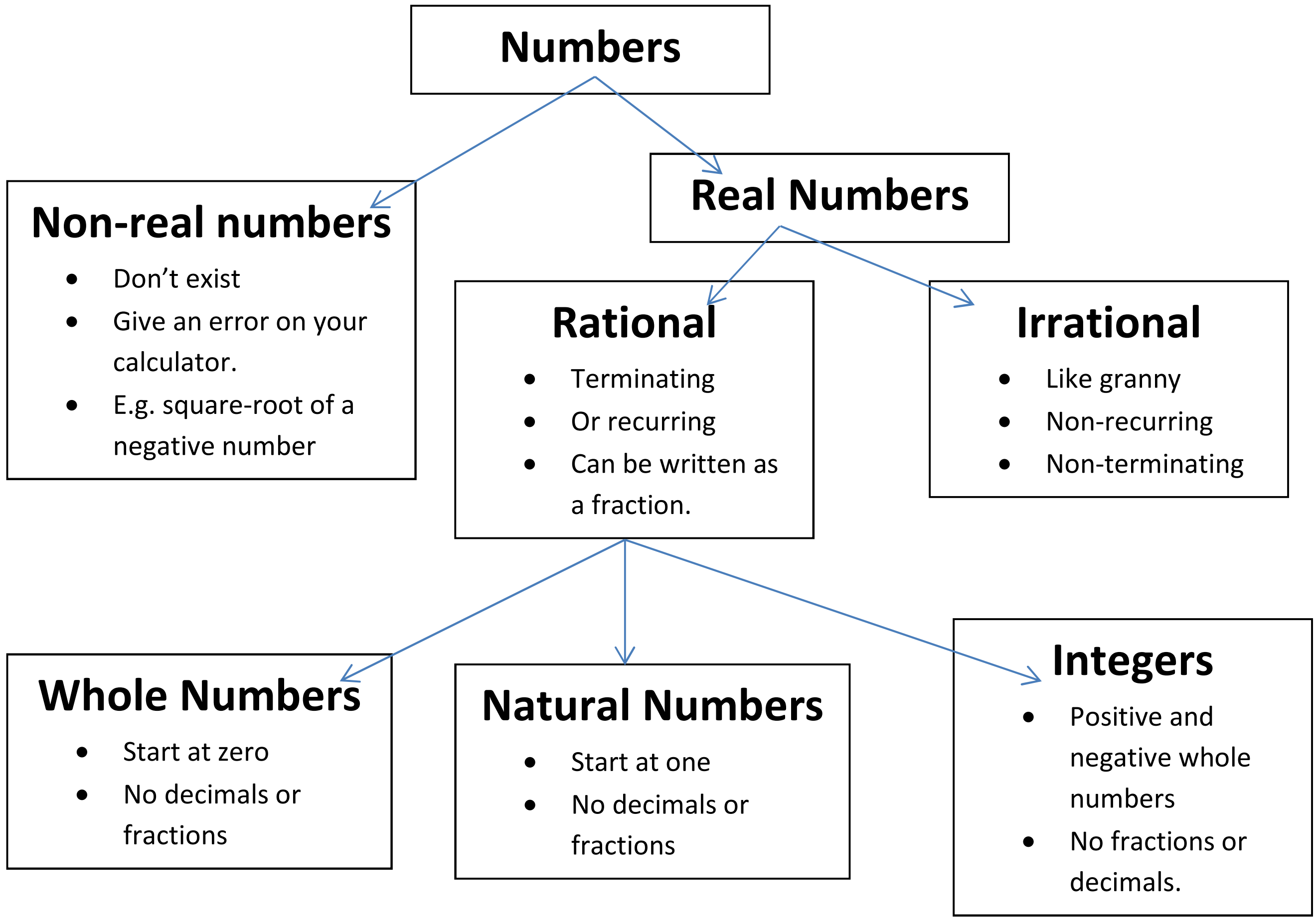
Numbers

Before you can start anything in mathematics – you need to know how numbers work. So here are the basics… (There is a summary on the next page – if you prefer flow diagrams)

Numbers are broken down into Real and Non-Real numbers

- Non-Real (R') numbers are numbers that don't exist (imaginary or complex numbers) – for example: the square-root of a negative number like $\sqrt{-3}$. These numbers will give an error on the calculator (On the SHARP El-535 the calculator will say – Error 2 – Calculation).
- Real Numbers (R) are numbers that do exist. They are broken down into Rational and irrational numbers
    - Irrational Numbers (Q') are numbers that don't make sense. I like to pretend they are like my grandmother – she likes to talk and talk but she doesn't make any sense. In the same way, irrational numbers go on and on (called non-terminating) and they don't have a pattern or make sense (called non-recurring). An example of an irrational number is $\sqrt{7} = 2{,}645751311 \ldots$ - do you see that it doesn't have a pattern and it doesn't stop?
    - Rational numbers on the other hand are either recurring (that means they have a pattern) for example 0,333333… or they are terminating (that means they end) for example 2,25 or 3. Rational numbers can be written as a fraction. Rational numbers are also broken down into:
        - Whole numbers (N_0) – these are numbers that start at zero (0) and count up in wholes, e.g. 0, 1, 2, 3… etc. They DO NOT have decimals or fractions. A nice way to remember whole numbers start with zero is that there is an **O** in wh**O**le.

- Natural numbers (N) – these are counting numbers which means that they start at one (1) and count up in wholes, e.g. 1, 2, 3… etc. They do NOT have decimals or fractions. A nice way to remember that natural numbers start with one is that there is a **1** in Natura**1**.
- Integers (Z) are positive and negative whole numbers, e.g. … -3, -2, -1, 0, 1, 2, 3… etc. They also do NOT have fractions or decimals.



⁎ *remember to learn both the names and the symbols – because a question can use either the symbols or the names.*

Activity 1

1. Tick the correct columns in the table below:

Number	Real	Non-Real	Rational	Irrational	Whole	Natural	Integer
0							
π							
-1							
-1,45							
$\sqrt{3}$							
$\sqrt{-8}$							
$\frac{13}{16}$							
$\left(\sqrt{-5}\right)^2$							

2. Given the equation: $0 = (x+3)(2x-5)(x^2+6)$

   Solve for x when x is

   a) real

   b) non-real

   c) an integer.

Rounding Off

Part of knowing your numbers is remembering the facts about rounding off – so here is some quick revision for you:

When you round off check how many places you want to round off to, then look at the number next to the last number you will have once you have rounded off →

e.g. round off to three decimal places: 0, 123456

place rounding off to

number you use to decide whether the number next door goes up or stays the same. If the number is 5 or bigger than 5 (that is 6, 7, 8, 9) then the number next door goes up. If the number is smaller than 5 (that is 4, 3, 2, 1, 0) the number next door stays the same.

* However, remember that when you are working with numbers representing people or things that cannot be a fraction be careful how you round up – think carefully about what the question is asking you for. For example, if you work out you have to cater for 43,2 people you would have to round UP to 44 because you cannot not cater for the 0,2 part of a person.

Also – when they say round off to the nearest unit – remember that a unit is a whole number.

Activity 2

1. Round off the following numbers to 2 decimal places:
   a) 0,135 b) 2,369
   c) 5,895 d) 4,351
2. Would you round the following up or down?
   a) the average number of dogs per house is 2,3
   b) the average number of people who live in a square meter in china is 11,6
   c) your bill at the supermarket comes to R11,97
   d) you worked for 2,13 hours
   e) the average number of children in a class is 34,3

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Surds between Integers

A surd is a root of a number that cannot be simplified any further, for example $\sqrt{3}$.

It is always good to estimate the answer of a surd so that you know that your calculator computation was correct – it's a way of checking yourself.

The easiest way to check is to figure where the surd lies on the number line.

$\sqrt{3} = 1{,}732 \ldots$ which shows us that it lies between 1 and 2.

But to work out where it lies without using a calculator you do this:

We count up in perfect squares → $1^2 = 1; 2^2 = 4;\ \ 3^2 = 9;\ \ 4^2 = 16;\ \ 5^2 = 25$ ....etc.

From this we can see that 3 lies between 1 and 4 on the perfect square line. Then we reverse it to see that $\sqrt{3}$ lies between 1 and 2 (the square-roots of the perfect square).

Ok… let's practice that again. Between which two integers does $\sqrt{18}$ lie?

Count up in squares: 1, 4, 9, 16, 25, 36…

We can see that 18 falls between 16 and 25.

Let's take the square-root of 16 and 25 to give us 4 and 5, so we know that $\sqrt{18}$ lies between 4 and 5.

Remember that if the question had said between $-\sqrt{18}$ our answer would lie between -5 and -4.

<u>Activity 3</u>

1. Between which two integers do the following surds lie?

a) $\sqrt{56}$	b) $-\sqrt{12}$
c) $-\sqrt{78}$	d) $\sqrt{15}$
e) $-\sqrt{43}$	f) $-\sqrt{29}$
g) $\sqrt{99}$	h) $\sqrt{8}$

2*. Between which two integers do the following surds lie?

a) $\sqrt[3]{6}$	b) $\sqrt[3]{-45}$
c) $\sqrt[3]{78}$	d) $-\sqrt[3]{130}$

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Exponents

$$\left.\text{Base}^{\text{exponent}}\right\} \text{power}$$

Let's revise the basic laws of exponents.

- Law 1: If you have the same bases and you multiply the power you add the exponents
  - $a^m \times a^n = a^{m+n}$
- Law 2: If you have the same bases and you divide the powers, you subtract the exponents.
  - $a^m \div a^n = a^{m-n}$
- Law 3: If you raise an exponent to another exponent you multiply the two exponents together.
  - $(a^n)^m = a^{n \times m}$
  - Remember that all the powers in the bracket are affected by the outside exponent: $(a^n b^m c)^p = a^{np} b^{mp} c^p$
- Law 4: If you take a root of a power the exponent inside is divided by the type of root.
  - $\sqrt[d]{a^c} = a^{\frac{c}{d}}$
  - A nice way to remember this rule is that cats live inside the house (c), dogs live outside the house (d) and cats are more important than dogs so they go on top.
- Law 5: Anything to the power of zero is one
  - $a^0 = 1$
- Law 6: If your power has a negative exponent the power goes under 1 and the exponent becomes positive (it also works in reverse – if you have a power under 1 with a negative exponent the power goes back on top and the exponent becomes positive).
  - $a^{-n} = \frac{1}{a^n}$ $\quad OR \quad$ $\frac{1}{a^{-n}} = \frac{a^n}{1}$ (but you can just write it as a^n).
  - A nice way to remember this is to think of the exponent as the girl/boyfriend of the base, when the exponent is negative the girl/boyfriend is negative and wants to go downstairs (or upstairs if the power is under the line). Once the power is downstairs (or upstairs) it makes the girl/boyfriend happy or positive.

You need to learn these exponent laws as they will help you later on with other work that you will cover all the way to matric. Learn them now and you won't be confused later ☺

There are two things you need to be able to do with exponents:

* solve
* and simplify

To simplify use your exponents to get to the simplest possible form of an answer (make sure that all your exponents are positive), for example, simplify:

1. $\frac{a^2b^3}{c^3} \times \frac{a^3c^5}{b^7}$ First multiply top with top and bottom with bottom (when you × you +)

$= \frac{a^{2+3}b^3c^5}{c^3b^7}$ Then divide like bases (when you ÷ you -)

$= a^5b^{3-7}c^{5-3}$

$= a^5b^{-4}c^2$ Finally write your answer with positive exponents

$= \frac{a^5c^2}{b^4}$ and now we can't simplify anymore so this is our final answer.

2. $\frac{49^{x+2}875^x}{35^{3x-1}}$ First break down the numbers into their prime factors

$49 = 7 \times 7 = 7^2$ $875 = 125 \times 7 = 5 \times 5 \times 5 \times 7 = 5^3 \times 7$

$35 = 5 \times 7$

$= \frac{(7^2)^{x+2}(5^3\times7)^x}{(5\times7)^{3x-1}}$ Multiply out the exponents

$= \frac{7^{2x+4}\ 5^{3x}7^x}{5^{3x-1}7^{3x-1}}$ Simplify by adding (when you multiply the bases) and subtracting

$= 7^{2x+4+x-(3x-1)}5^{3x-(3x-1)}$ (when dividing).

$= 7^{3x+4-3x+1}5^{3x-3x+1}$

$= 7^5 5^1$

$= 84\ 035$

To solve you will be given an equation and you need to work out what the value of x is. There are three different ways of asking for x:

* x as an exponent
* x as a base
* x as an answer.

Here are some examples:

Solve for x:

1. $3.2^x - 5 = 7$ First take the number that is not attached to the power to the other side.

> Prime factors are factors of a number that are prime (they only have the factors 1 and itself). E.g. to prime factorise 48 – break 48 into factors – 6 x 8 and then further break down those numbers into 3 x 2 and 4 x 2. 3 and 2 are prime numbers so we can't factorise them anymore. The only number is 4 which is 2 x 2. So 48 prime factorised is 3x2x2x2x2 or 3×2^4

$3.2^x = 12$ Then divide by the 3.

$2^x = 4$ Now prime factorise the number.

$2^x = 2^2$ Because the bases are the same we know the exponents are the same, so....

$x = 2$

2. $5^{x+1} - \frac{5^{x+1}}{2} = 312\frac{1}{2}$ Separate the 5^{x+1} into two separate bases.

$5^x 5^1 - \frac{5^x 5^1}{2} = 312\frac{1}{2}$ Now take out the 5^x as a common factor

$5^x\left(5 - \frac{5}{2}\right) = 312\frac{1}{2}$

$5^x\left(2\frac{1}{2}\right) = 312\frac{1}{2}$ Divide both sides by $2\frac{1}{2}$

$5^x = 125$ Prime factorise the 125.

$5^x = 5^3$ Drop the bases on both sides

$\therefore x = 3$

3. $\frac{x^3}{4} + 4 = 2\frac{3}{4}$ First take the number that is not attached to the power across.

$\frac{x^3}{4} = 6\frac{3}{4}$ Now multiply out by the 4

$x^3 = 27$ Then take the cube-root of both sides

$\sqrt[3]{x^3} = \sqrt[3]{27}$ (a cube-root of a cube cancels the other out).

$\therefore x = 3$

4. $4^5 = x$ Simply put the power into your calculator by pressing:

$\therefore x = 1\,024.$

Activity 4

1. Simplify the following:

a) $\dfrac{c^4d^2e}{f^2d^{-2}} \times \dfrac{(e^3f)^{-1}}{\sqrt[3]{d^{-3}c^6}}$

b) $\dfrac{1}{a^{-2}} \times \left(\dfrac{b}{a^3}\right)^{-1} \div \dfrac{b^2}{a^2}$

c) $\dfrac{x^{-\frac{1}{4}}y^3}{z^3} \times \dfrac{(yz)^{-3}}{\sqrt[4]{x^5}}$

d) $\dfrac{18^x.24^{x+1}}{36^{2x-1}}$

e) $\dfrac{21^{x+1}.63^{x-1}}{9^x.49^{2x}}$

f) $\dfrac{a^{-1}b^6}{c^{-6}d^4} \times \dfrac{ab^4c^5}{d^{-7}} \div \dfrac{\left(a^3b^5\right)^2}{c^{-1}d^3}$

g) $\dfrac{(ef^{-1})^4}{g^{-2}h^3} \times \dfrac{e^6f^4g^2}{h} \times \left(\dfrac{e}{f^{-2}}\right)^{-1}$

h) $\left(\dfrac{1}{x} + \dfrac{x}{y}\right)^{-2}$

i) $\left(\left(\dfrac{x}{y}\right)^{-2} + \left(\dfrac{x}{y} - \dfrac{y}{x}\right)^{-3}\right)^0$

j) $\dfrac{7^{x+1} \times 2^{2x-3}}{14^{x-5}}$

2. Solve for x:

a) $2^x - \dfrac{2^{x+1}}{5} = 0{,}3$

b) $2^{x+1} - \dfrac{2^{x+1}}{3} = 21\dfrac{1}{3}$

c) $10x^3 + 50 = 1\ 300$

d) $\dfrac{4x^4}{3} - 5 = 103$

e) $6^{2\frac{1}{2}} = x$

f) $x = 4^{1\frac{1}{3}}$

g) $4^{x+2} - 4^x = \dfrac{15}{16}$

h) $4^x - \dfrac{4^x}{2} = 1$

i) $5^x + 3.5^x = 4$

j) $-\dfrac{1}{5}x^3 + \dfrac{3}{5} = -1$

Multiplying Algebraic Expressions

When you multiply a single term with a bracket (where there could be many terms) each term in the bracket is multiplied by that single term.

When you have a bracket with more than one term, being multiplied with a bracket that has more than one term you need to make sure that each term in the first bracket is multiplied by each term in the second bracket. An easy way to check that you are doing this is to draw arrows in one colour to each term in the second bracket, then do the next term in another colour and so on...

For example, multiply $2x - y$ by $3x^2 - 4xy + y^2$

1 2 3

$(2x - y)(3x^2 - 4xy + y^2)$

4 5 6

Start with the $2x$ and multiply each term in the second bracket with the $2x$ (arrows in blue). Then do the second term, $-y$ (arrows in green). Don't forget to multiply out the signs as well.

1 2 3 4 5 6

$6x^3 - 8x^2y + 2xy^2 - 3x^2y + 4xy^2 - y^3$

$6x^3 - 11x^2y + 6xy^2 - y^3$

Now simplify your answer by adding all the like terms together.

Activity 5

Multiply out and simplify the following expressions

a) $(3x - 5)(x^2 + 4x - 6)$

b) $\left(\frac{1}{2}y + 2\right)(4y^2 - 6y + 3)$

c) $\left(\frac{1}{4}x - 1\right)\left(-x^2 - \frac{1}{3}x + 4\right)$

d) $(6x + 4y)(2x^2 + 4xy + 3)$

e) $(-4x + 3)\left(\frac{1}{5}x^2 + 11x - 6\right)$

f) $(-3a - 2b)(-7a^2 + 8ab - b^2)$

g) $\left(\frac{3}{4}c^2 - 3d\right)\left(\frac{1}{2}c^2 + cd + 6d^2\right)$

h) $(5e + 2f)\left(e^2 - 8ef + \frac{1}{3}f^2\right)$

i) $(-2g + 5h)(6g - 6gh + h)$

j) $\left(8k - \frac{4}{5}m\right)\left(2k^3 + \frac{1}{4}k^2m - km\right)$

Factorising

Factorising is taking many terms and making them into one term. Factorising will be used throughout the rest of your maths career so make sure you understand this section.

There are four steps to factorisation:

1. Look for a highest common factor (if there is one take it out)

   e.g. $2x^2 - 8 \rightarrow$ the 2 is the highest common factor because it goes into both terms. When you take it out (in other words you divide each term by 2 and leave the 2 on the outside of the brackets and the answers inside the brackets) and put it in the front you get $2(x^2 - 4)$. Remember that if you multiply out the brackets you have just factorised it should give you your original equation or expression.
2. Now look at how many terms the expression has
   a. If there are two terms it could be
      i. A difference of squares
         1. To factorise: $a^2 - b^2$ say
            $\left(\sqrt{first\ term} - \sqrt{second\ term}\right)\left(\sqrt{first\ term} + \sqrt{second\ term}\right)$
      ii. A difference or sum of cubes
         1. To factorise: $a^3 \pm b^3$ say:

$$\left(\sqrt[3]{first\ term} \pm \sqrt[3]{second\ term}\right)\left(\left(\sqrt[3]{first\ term}\right)^2 \mp \sqrt[3]{first\ term} \times \sqrt[3]{second\ term} + \left(\sqrt[3]{second\ term}\right)^2\right)$$

   b. If there are three terms it could be a trinomial.
      i. The easiest way to factorise a trinomial is to use your calculator, for example, factorise $x^2 - 5x - 24$. (Some theory before I show you how: remember after you have factorised you will have to brackets – if you multiply them out you will get two x terms which you add (whether the sign is minus or plus) together to give you the middle term. We are going to use this fact in our method).

      To find factor pairs go to table mode by pressing **MODE** **3**

Then enter your “c” or constant value (e.g. -24) by typing (−) NEG 24 then press a/b to put it at the top of a fraction. At the bottom enter an X by pressing RCL twice. Press the = ENTER button.

This will take you through to a screen that says:

X_Start: 0

X_Step: 1

Press = ENTER twice.

Now you will see a table with different factor pairs

X	ANS
0	- - - - - -
1	- 24
2	- 12

Look at your factor pairs and add each set together, e.g. 1 + -24 = -23. This is not our middle term so we look at the next factor pair 2 and -12. We continue until we find a factor pair that gives us -5, in this case 3 and -8. So now we know our brackets will be $(x+3)(x-8)$.

If there is a number in front of your x^2 divide the entire trinomial by that number (or take it out as a common factor). Then follow the same steps above but make your step a fraction (1 over the common factor you took out).

E.g. Factorise: $2x^2 - x - 6$

1. Take 2 out as a common factor: $2\left(x^2 - \frac{1}{2}x - 3\right)$
2. Go into table mode (MODE 3) and then type in (−) NEG 3 a/b RCL RCL = ENTER
3. For X_Start press = ENTER to leave it at 0.
4. For X_Step press 1 a/b 2 = ENTER
5. Now look at the table and find the factors pairs that add up to $-\frac{1}{2}$

6. In our case the pair is $1\frac{1}{2}\ and - 2$

7. so the brackets will look like this: $2\left(x+1\frac{1}{2}\right)(x-2)$

8. Multiply your common factor back into the first bracket (or the bracket with the fraction) to complete your factorising:
$(2x+3)(x-2)$

c. If there are four terms it means you need to group terms that have the same variable together and then take a common factor out of each group.

e.g. $3a^2+3ab+2a+2b$

As you can see – this is already grouped together (3's together and 2's together).

Take a common factor from the first two terms (3a) and a common factor from the next two terms (2).

$3a(a+b)+2(a+b)$

Now you can see that the brackets for each group are the same, so we can take these brackets out as a common factor and put the original common factors in a bracket together behind the "new" common factor bracket.

$(a+b)(3a+2)$

Activity 6

1. Factorise the following by taking out a common factor:

a) $2a^2+4ab-8a^3b^2$ b) $5c^6d^6-30c^4d^2+85c^5d^3$

c) $72ab^2c\ +48bc^2-36ac$ d) $9xy+3x^2y-12x$

2. Factorise the following cubes and squares:

a) x^2-y^2 b) $2x^2+2y^2$

c) $8x^3-y^3$ d) $125+343g^3$

e) $54a-16a^4$ f) $16c^2-25d^2$

g) $216b^3+8a^3$ h) x^3y-y^4

i) $49e^3-64f^2e$ j) $50x^4-800y^4$

3. Factorise the following trinomials:

a) $x^2 - x - 30$
b) $x^2 + 9x + 20$
c) $x^2 + 13x + 42$
d) $x^2 - 7x + 6$
e) $x^2 + x - 12$
f) $x^2 - 2\frac{1}{5}x + \frac{2}{5}$
g) $3x^2 - 7x - 6$
h) $2x^2 - 11x + 5$
i) $4x^2 - 19x - 5$
j) $6x^2 + 7x - 5$
k) $3x^2 - 14x + 8$
l) $3x^2 - 5x + 2$
m) $2x^2 - 11x - 21$
n) $2x^2 - 27x + 81$
o) $2x^2 + 3x - 5$
p) $4x^2 + 20x + 25$

4. Factorise the following by grouping:

a) $8ab - 12a^2b - 20b^2 + 30ab^2$
b) $c^3 + 18d^2 - 6cd - 3c^2d$
c) $-12ab^2 - 18a^4b^2 + 20a^2b + 30a^5b$
d) $2e^2f^3 - 6e^2f - 3ef^3 + 9ef$
e) $16g^3 - 64g^2h - gh + 4h^2$
f) $2j^3 - 12k^3 + 3kj - 8j^2k^2$
g) $3m^3 - 14n^4 + 21mn - 2m^2n^3$
h) $q^3 - 3p^2q - 8q^2p + 24p^3$
i) $7r^3 - 56r^2t + 10rt - 80t^2$
j) $v - 15v^2w^2 + 5v^3w - 3w$

5. Factorise the following: (use any of the above methods)

a) $2x^2 - 49x - 25$
b) $x^2 - 6x + 9$
c) $80x^3 - 500xy^2$
d) $6x^3 - 48y^3$
e) $13a^3 - 30b^3 - 78ab + 5a^2b^2$
f) $4x^2 - 11x + 6$
g) $3x^2 + 6x - 24$
h) $\frac{1}{25}x^2 - 16y^4$
i) $6x^2 + 5x - 4$
j) $a^3b^6 + 125c^3$
k) $10x^2 - 3x - 18$
l) $\frac{1}{3}x^2 - 1\frac{2}{3}x + 2$
m) $8a^3 - 42b^3 - 6a^2b^2 + 56ab$
n) $0{,}01x^2 + y^2$
o) $2x^2 - \frac{1}{2}x - \frac{1}{4}$
p) $6x^2 + 43x + 65$
q) $2x^2 - 9x - 110$
r) $-81x^4 - 24xy^3$
s) $a^3 - 7ab + 112b^3 - 16a^2b^2$
t) $x^4 - 18x^2 + 32$
u) $3x^2 + 7x - 40$
v) $4x^2 - 5x - 6$
w) $16x^4 - y^4$
x) $x^6 + y^6$

Algebraic Fractions

Now don't skip this section just because the heading says algebraic fractions. Fractions are actually pretty straight forward if you remember the basics.

So here are the basics:

- Numerators are the numbers and variables at the top of the fraction, denominators are the numbers and variables at the bottom of the fraction.

  $$\frac{numerator}{denominator}$$

- The Golden rule: What you do to the top (e.g. multiply by two) you do to the bottom (also multiply by two) → fairs fair.
- Before you can add and/or subtract fractions, you need to make sure the denominators are the same, e.g. $\frac{1}{2}+\frac{1}{3}\rightarrow$ the LCD (lowest common denominator) is 6. How did we get that? By multiplying the two denominators together (2 x 3 = 6). In the same way for $\frac{1}{a}+\frac{1}{b}$ the denominator is *ab* (because *a* x *b* = *ab*).
- You can never ever cancel over two terms (in other words – don't cancel over a plus or minus). If there are too many terms try factorising first.

  e.g. $\frac{x^2+4}{x}\rightarrow$ you **cannot** cancel the x's because there are two terms on the top.

  e.g. $\frac{x^2+4x}{x}\rightarrow$ first take out a common factor of x at the top $\rightarrow \frac{x(x+4)}{x}$ (Now you have one term over one term – remember the definition of factorising) and so you **can** cancel so your answer will now be $x+4$.
- Remember to try to simplify before you add, subtract, multiply or divide – this will save you lots of time later on.
- Your denominator can **NEVER** be zero (0). This will cause your entire sum to be undefined.
- Also remember that you can take out a negative 1 as a factor.
- When you multiply fractions you multiply top with top and bottom with bottom.
- When you are dividing fractions remember to turn the fraction up-side-down and then multiply, e.g. $\frac{1}{a}\div\frac{b}{c}\rightarrow\frac{1}{a}\times\frac{c}{b}=\frac{c}{ab}$

So let's try some examples:

1. $\frac{x+1}{x^2-1}+\frac{x+2}{x^2+x-2}$

First factorise and simplify so that you can see all the possible factors that could go into your common denominator.

$=\frac{(x+1)}{(x-1)(x+1)}+\frac{(x+2)}{(x-1)(x+2)}$

You can see that from the first fraction the $x+1$ in the numerator and the denominator cancel and the $x+2$ in the numerator and the denominator of the second fraction cancel.

$=\frac{1}{x-1}+\frac{1}{x-1}$

Now you can see that the denominators are the same, so you can add the numerators together.

$=\frac{2}{x-1}$

2. $\frac{x^3+y^3}{2x+2y}\times\frac{4x+6y}{x^2-xy+y^2}$

Always factorise first

$=\frac{(x+y)(x^2-xy+y^2)}{2(x+y)}\times\frac{2(2x+3y)}{(x^2-xy+y^2)}$

Now see if you can cross cancel → you can only cross cancel if there is a multiply sign between the two fractions.

$=\frac{\cancel{(x+y)}\cancel{(x^2-xy+y^2)}}{\cancel{2}\cancel{(x+y)}}\times\frac{\cancel{2}(2x+3y)}{\cancel{(x^2-xy+y^2)}}$

Do you see the three sets of "common" factors that you can take out?

$=\frac{1}{1}\times\frac{(2x+3y)}{1}$

Now multiply top with top and bottom with bottom.

$=2x+3y$

3. $\frac{a^2+ab}{3ab}\div\frac{a^2-b^2}{6b}$

Always factorise first

$=\frac{a(a+b)}{a(3b)}\div\frac{(a+b)(a-b)}{6b}$

Remember that you cannot cross cancel across a divide sign so first turn the fraction up-side-down.

$=\frac{a(a+b)}{a(3b)}\times\frac{6b}{(a+b)(a-b)}$

Now you can cancel

$=\frac{\cancel{a}\cancel{(a+b)}}{\cancel{a}\cancel{(3b)}}\times\frac{\cancel{6b}\,2}{\cancel{(a+b)}(a-b)}$

Notice that 3 goes into 6 twice so there is a remainder of 2 at the top.

$=\frac{1}{1}\times\frac{2}{a-b}$

Now you can multiply

$=\frac{2}{a-b}$

4. $\frac{x^3-y^3}{x^2-y^2}+\frac{2x+y}{x-y}$ First factorise the expression

$=\frac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)}+\frac{2x+y}{x-y}$ Cancel anything you can.

$=\frac{x^2+xy+y^2}{x+y}+\frac{2x+y}{x-y}$ Find the LCD – in this case $(x+y)(x-y)$ and multiply

$=\frac{(x-y)(x^2+xy+y^2)+(x+y)(2x+y)}{(x+y)(x-y)}$ Multiply out and simplify.

$=\frac{x^3+x^2y+xy^2-yx^2-xy^2-y^3+2x^2+xy+2xy+y^2}{(x+y)(x-y)}$

$=\frac{x^3+2x^2+3xy+y^2-y^3}{(x+y)(x-y)}$ You cannot factorise or simplify any further so your sum is complete.

<u>Activity *7*</u>

Simplify the following fractions assume that all denominators do not equal zero:

a) $\frac{5x^2-20y^2}{5xy}\times\frac{2xy^2}{5x+10y}$

b) $\frac{a}{a-b}-\frac{b}{b-a}$

c) $\frac{p}{3p-6q}+\frac{2p-4q}{p^2-4q^2}$

d) $\frac{x^3-8y^3}{x^2-4y^2}\div\frac{x^2+2xy+4y^2}{6x+12y}$

e) $\frac{m+2n}{2mn}+\frac{m}{2m-4n}-\frac{3m+1}{2n-m}$

f) $\frac{x^2-4}{x^3-8}\times\frac{2-x}{x+2}$

g) $\frac{a+b}{a^2-b^2}\times\frac{6a^2b}{3a+6b}\div\frac{2ab}{a^3-b^3}$

h) $\frac{a}{a+b}+\frac{1}{b-a}\times\frac{a^2-b^2}{2a-3b}$

i) $\frac{x^2y}{3x^3+81y^3}\div\frac{x+2y}{3x^4-48y^4}\times\frac{x+3y}{x-2y}$

j) $\frac{3}{a+b}-\frac{2}{a^2-b^2}+\frac{5}{b-a}$

k) $\frac{-2(x+y)}{x^3-y^3}+\frac{1}{x-y}-\frac{6}{x^2+xy+y^2}$

l) $\frac{8x^3+125}{x^2+4x+4}\times\frac{-2-x}{x+5}\times\frac{4x+8}{2x+5}$

m) $\frac{v^2+vw}{v^2-w^2}\times\frac{w-v}{2v+4w}\div\frac{5v+8w}{4w+2v}$

n) $\frac{1}{a+b}+\frac{3}{a-b}-\frac{2a+b}{b^2-a^2}$

****

Solving Equations

What is the difference between an expression and an equation?

An expression does not have an equal sign in the middle – you can only simplify an expression, whereas an equation has an equal sign in the middle and you can solve for a variable because the one side is equal to the other side.

This is an expression: $2(x + y^2)$

And this is an equation: $x - 5 = 2x - 7$

Do you see that the expression can be simplified to $2x + 2y^2$ and then cannot be changed or manipulated anymore?

However – we can manipulate the equation so that we can actually find the value for x.

$$x - 5 = 2x - 7$$
$$x - 5 - x = 2x - 7 - x$$ (What you do to one side you do to the other side)
$$-5 + 7 = x - 7 + 7$$
$$2 = x$$

Do you remember that whatever you do to one side you HAVE to do to the other side? (Fairs Fair – if your mom bought you an ice-cream but not your sister an ice-cream that wouldn't be fair – in the same way you can't do one thing on the one side and not do it on the other side because that wouldn't be very fair would it? In fact you would actually be changing the equation and your final answer would be wrong).

There are several different types of equations:

- Linear – when there are **NO** exponents in the equation, e.g. $2x + 4 = x - 5$
    - Its very easy to solve these kinds of equations, you simply take all the x's to one side and the numbers to the other side and then find the value for one x.
    - E.g. $2x + 4 = x - 5$ Take the x to the left by subtracting it from both sides.

      $2x + 4 - x = x - 5 - x$

      $x + 4 = -5$ Now take the 4 to the right

      $x + 4 - 4 = -5 - 4$

      $x = -9$ (When you are more confident you can do the two steps together- just be careful not to make a mistake).

- Quadratic – these are equations in the form $ax^2 + bx + c$ where *a, b*, and *c* are constants (numbers without variables) or any equation where the highest power of x is x^2.
    - To solve quadratic equations you need to factorise the equation and make each factor equal to zero. Then you solve for each factor.
    - E.g. $x^2 + x = 30$ First take everything over to the left so that you only have a zero on the right.

      $x^2 + x - 30 = 0$ Now you can factorise.

      $(x+6)(x-5) = 0$ Now we make each factor (the part in brackets) equal to zero.

      $\therefore x + 6 = 0 \quad or \quad x - 5 = 0$ Now solve for x individually

      $\therefore x + 6 - 6 = \ -6 \qquad x - 5 + 5 = +5$

      $\therefore x = \ -6 \quad or \qquad x = 5$
    - An easy way to remember that you need two answers is that there is a 2 on the x^2.

- Simultaneous Equations – these are two equations that are happening at the same time on the same Cartesian plane. You have to figure out where the two "graphs" or equations will intersect. In other words, where are the two equations equal to each other? This means that the coordinates (or x and y values will be the same for both graphs / equations).
    - There are two methods for solving simultaneous equations.
        - The first is substitution: you need to find either an x or a y by itself and then substitute it into the other equation. Then you will only have one variable that you are solving for. Once you find that variable, substitute it back into the first equation to find the other variable's value. (You can use this in any kind of simultaneous equations)
        - The second method is elimination: subtract the one equation from the other equation so that one of the variables is cancelled out. You may need to multiply one of the equations first so that the one variable has the same constant. Then solve for the remaining variable. Once you have the answer you can substitute it back into one of the original equations to find

the value of the other variable (use this method only if the two equations are of the same type – e.g. both are linear or both are quadratic).

- E.g. Solve for x and y if $y = 3x + 5$ and $2y = 7x - 3$
  - Method 1: Substitution
    - Get either $x \; or \; y$ by itself (already done for you).
    - Next Substitute it into the other equation:

      $y = 3x + 5 \qquad \rightarrow 2(3x + 5) = 7x - 3$
    - Now solve that equation for x

      $6x + 10 = 7x - 3$
      $6x + 10 - 10 - 7x = 7x - 3 - 7x - 10$
      $-x = -13$
      $\therefore x = 13$
    - Now that you have x substitute it back into $y = 3x + 5$ to find the value of y.

      $y = 3(13) + 5$
      $\therefore y = 39 + 5$
      $\therefore y = 44$
  - Method 2: Elimination
    - In this equation we will eliminate the y first. This means that we need to multiply equation 1 by two so that the y's have the same constant.

      $2 \times (y = 3x + 5)$
      $2y = 6x + 10$ (Note the entire equation is multiplied).
    - Now subtract the one equation from the other (it doesn't matter which order you put them in).

      $2y = 6x + 10$
      $-2y = -7x + 3$
      $0 = -x + 13$ Now solve for x
      $+x = -x + 13 + x$
      $\therefore x = 13$
    - Now we substitute it back into one of the original equations to find the value of y

      $\therefore y = 3(13) + 5$
      $\therefore y = 44$

- Word Problems – yes, that means story sums ☺ All you have to do is read the story very carefully and underline all the important information. Once you've done that put the information together – either in linear equations or in a table format. From there you can form the equations in order to solve for the variables. They can use linear, quadratic and simultaneous equations in word problems so pay special attention to how things are said.

- Literal equations – these are equations where you have to make one of the variables the subject of the formula (in other words, get that variable by itself and take all the other variables and numbers to the other side).
    - E.g. Solve for r in terms of v, h and t

$V = \frac{rh}{t} + 3$	First take the 3 to the other side
$V - 3 = \frac{rh}{t} + 3 - 3$	
$V - 3 = \frac{rh}{t}$	Then take the t to the other side
$(V-3) \times t = \frac{rh}{t} \times t$	
$t(V-3) = rh$	Then divide the rh by h to get r by itself
$t(V-3) \div h = \frac{rh}{h}$	
$\frac{t(V-3)}{h} = r$	Now you have r by itself so you have answered the question.

- Inequalities – these involve <, >, ≥ and ≤ signs. They work exactly like an equals sign with two exceptions:
        - When you divide or times by a negative number your inequality sign flips over.
        - You can never divide or multiply across the inequality sign by the variable you are looking for because you do not know the sign (whether its positive or negative) of the variable.
    - When you are solving for an inequality you need to be able to draw a graph or representation of the inequality on a number line. Here are a couple of rules to follow:
        - < > are represented by open dots ○

- $\leq \geq$ are represented by closed dots 
- Real numbers are drawn with a solid line
- Integers are drawn with closed dots between the two values of the solved inequality – do you remember why?
- Whole numbers start from zero and are drawn with closed dots between the two values of the solved inequality
- Natural numbers start from one and are drawn with closed dots between the two values of the solved inequality.

o E.g. Solve for x if $x \in R$ and represent the answer graphically :

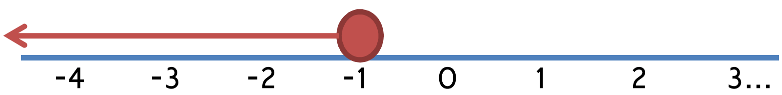
$-2x + 3 \geq 5$

$-2x + 3 - 3 \geq 5 - 3$ Take the constant to the other side

$-2x \geq 2$

$-\frac{2x}{-2} \geq \frac{2}{-2}$ Divide by -2 → Remember because you are dividing by a negative the inequality sign flips over.

$\therefore x \leq -1$



o Interval Notation – this is another way of writing inequalities. Sometimes you will be asked to give your answer in interval notation.

- $<$ $>$ are represented by (and)
- $\leq \geq$ are represented by [and]
- Put the bracket first then the smallest number then “;” and then the biggest number followed by the second bracket.
- E.g. $2 < x \leq 5$ $\rightarrow x \in (2; 5]$

Activity 8

1. Solve for x in the following linear equations:

a) $2x - 1 = -9x + 6$

b) $6x - 1 = 3x - 2$

c) $\frac{1}{2}x - 10 = 4x + 4$

d) $-5x + 1 = 4x - 8$

e) $3x - 5 = x + 13$

f) $-2x - 5 = 7x + 2$

g) $-\frac{1}{2}x - \frac{1}{6} = 5x + 1\frac{5}{6}$

h) $3(x + 5) = 3x + 9$

i) $x - 2 = 4(2x + 3)$

j) $x + 7 = 3(2x + 4)$

2. Solve the following quadratic equations for x:

a) $x^2 + 7x + 10 = 0$

b) $x^2 + 12x + 36 = 0$

c) $x^2 + 7x - 44 = 0$

d) $x^2 - 5x + 4 = 0$

e) $2x^2 - 13x + 15 = 0$

f) $x^2 - 16 = 0$

g) $2x^2 - 8 = 0$

h) $3x^2 - 5x + 2 = 0$

i) $5x^2 - 42x - 27 = 0$

j) $3x^2 = 16 - 8x$

3. Solve the following simultaneous equations for x and y:

a) $y = 2x - 3$ and $2y = 5x - 8$

b) $3y = -4x + 1$ and $2y = x - 4$

c) $3y = x$ and $y = 2x + 5$

d) $y + x = 1$ and $5y = -x + 3$

e) $-2y = 5x + 6$ and $y = 4x - 5$

f) $y = -5x$ and $2y = -2x - 1$

g) $5y = 2x + 8$ and $2y = x + 4$

h) $y = 4x + 5$ and $3x - y = 2$

i) $y = 2x - 4$ and $x - 8y = 27$

j) $6y - x = 0$ and $y = -5 + \frac{1}{3}x$

4. Solve the following word problems:

a) Sarah is 20 years younger than her dad. In 5 years' time she will be half of her dad's age. How old are Sarah and her dad now?

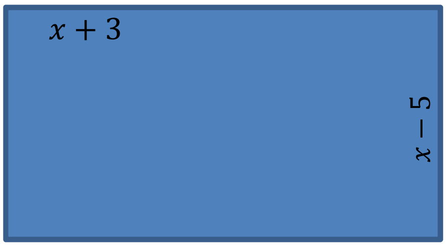
b) Joe cycles to school and back every day. It takes him twice as long on the way to school as on the way home. If Joe lives 10km from the school, determine how long it takes Joe to go to school and back home if his average speed is 25km/h.

c) Bob goes to the local café and buys 3 cokes and a packet of chips for R27. The next time he goes to the café he buys 2 cokes and 3 packets of chips and it costs him R25. Work out how much a coke costs and a packet of chips costs.

d) Dan wants to work out how much fencing he needs for his farm. He knows that the ground is a rectangular shape and that the breadth is equal to twice the length plus 1km. Determine the length of the rectangle and hence determine the length of fencing Dan needs if the area of the farm is 300km^2.

e) George knows that the perimeter of his fence is 100m around his rectangular piece of garden, and that the breadth is $x + 5$m. Determine the length of the garden given that the area is 400m^2.

f) Given the sketch of a wall below.



Determine the value(s) of x if the surface area of the wall is 33m^2.

g) Anne knows that an ice-cream cone can hold 47,1239cm^3 of ice-cream. If the formula for the volume of a cone is $v = \frac{1}{3}\pi r^2 h$ and Anne knows the height of the cone is 5cm, determine how wide her scoop of ice-cream can be.

5. For each of the following find the given variable in terms of the other variables:

a) Find r if $v = \pi r^2$

b) Find t if $s = \frac{d}{t}$

c) Find *i* if $A = P(1 + i)^n$

d) Find r if $v = \frac{4}{3}\pi r^3$

e) Find h if $v = \pi r^2 h$

f) Find m if $y = mx + c$

g) Find d if $T = a + (n - 1)d$

h) Find x if $y = -ax^2 + 3$

6. Determine the values for x and represent the answers both graphically and in interval notation.

a) $3x + 4 \leq 5x - 8$

b) $-2x + 5 > 6$

c) $2x + 5 \geq 5$

d) $x - 6 < 4$

e) $-4x + 1 \leq 2$

f) $5 - 6x \geq 2x$

g) $x - 6 > 9x - 2$

h) $-x + 3 \leq 2$

i) $x - 8 < 2x + 5$

j) $-2x + 3 > 10$

Answers for the Activities

Activity 1

1.

Number	Real	Non-Real	Rational	Irrational	Whole	Natural	Integer
0	✓		✓		✓		✓
π	✓			✓			
-1	✓		✓				✓
-1,45	✓		✓				
$\sqrt{3}$	✓			✓			
$\sqrt{-8}$		✓					
$\frac{13}{16}$	✓		✓				
$(\sqrt{-5})^2$		✓					

2. Given the equation: $0 = (x + 3)(2x - 5)(x^2 + 6)$

Solve for x when x is

a) real $\rightarrow$ $x = -3 \quad or \quad x = \frac{5}{2}$

b) non-real $\rightarrow$ $x = \pm\sqrt{-6}$

c) an integer $\rightarrow$ $x = -3$

Activity 2

1. a) 0,135 → 0,14 b) 2,369 → 2,37
   c) 5,895 → 5,90 d) 4,351 → 4,35

2. a) up b) up
   c) down d) down
   e) up

Activity 3

1. Before we start we should do a "square" number line first

   1 4 9 16 25 36 49 64 81 100

   a) between 7 and 8 b) between -4 and -3
   c) between -9 and -8 d) between 3 and 4
   e) between -7 and -6 f) between -6 and -5
   g) between 9 and 10 h) between 2 and 3

2. Before we start we should do a "cube" number line first:

   1 8 27 64 125 216 343...

   a) between 1 and 2 b) between -4 and -3
   c) between 4 and 5 d) between -6 and -5

Activity 4

1. a) $$\frac{c^4d^2e}{f^2d^{-2}} \times \frac{(e^3f)^{-1}}{\sqrt[3]{d^{-3}c^6}}$$
$$= \frac{c^4d^2d^2e}{f^2} \times \frac{e^{-3}f^{-1}}{d^{-1}c^2}$$
$$= \frac{c^4d^4e}{f^2} \times \frac{d}{c^2e^3f}$$
$$= c^{4-2}d^{4+1}e^{1-3}\,f^{-2-1}$$
$$= c^2d^5e^{-2}f^{-3}$$
$$= \frac{c^2d^5}{e^2f^3}$$

b) $$\frac{1}{a^{-2}} \times \left(\frac{b}{a^3}\right)^{-1} \div \frac{b^2}{a^2}$$
$$= \frac{a^2}{1} \times \frac{a^3}{b} \times \frac{a^2}{b^2}$$
$$= \frac{a^7}{b^3}$$

c) $$\frac{x^{-\frac{1}{4}}y^3}{z^3} \times \frac{(yz)^{-3}}{\sqrt[4]{x^5}}$$
$$= \frac{y^3}{x^{\frac{1}{4}}z^3} \times \frac{1}{x^{\frac{5}{4}}y^3z^3}$$
$$= \frac{y^3}{x^{\frac{6}{4}}y^3z^6}$$
$$= \frac{1}{x^{\frac{3}{2}}z^6}$$

d) $9 \times 2 = 3^2 \times 2$ $\quad$ $8 \times 3 = 2^3 \times 3$ $\quad$ $9 \times 4 = 3^2 \times 2^2$

$$\frac{18^x.24^{x+1}}{36^{2x-1}}$$
$$= \frac{(3^22)^x.(2^33)^{x+1}}{(3^22^2)^{2x-1}}$$
$$= \frac{3^{2x}2^x\,2^{3x+3}3^{x+1}}{3^{4x-2}2^{4x-2}}$$
$$= 3^{2x+x+1-(4x-2)}2^{x+3x+3-(4x-2)}$$
$$= 3^{3x+1-4x+2}2^{4x+3-4x+2}$$
$$= 3^{3-x}2^5$$
$$= 3^{3-x}32$$

e) 7×3 $\quad$ $7 \times 9 = 7 \times 3^2$ $\quad$ 3^2 $\quad$ 7^2

$$\frac{21^{x+1}.63^{x-1}}{9^x.49^{2x}}$$
$$= \frac{(7.3)^{x+1}(7.3^2)^{x-1}}{(3^2)^x.(7^2)^{2x}}$$
$$= \frac{7^{x+1}3^{x+1}.7^{x-1}3^{2x-2}}{3^{2x}.7^{4x}}$$
$$= 7^{x+1+x-1-(4x)}3^{x+1+2x-2-(2x)}$$
$$= 7^{2x-4x}3^{3x-1-2x}$$
$$= 7^{-2x}.\,3^{x-1}$$
$$= \frac{3^{x-1}}{7^{2x}}$$

f) $$\frac{a^{-1}b^6}{c^{-6}d^4} \times \frac{ab^4c^5}{d^{-7}} \div \frac{\left(a^3b^5\right)^2}{c^{-1}d^3}$$
$$= \frac{b^6c^6}{ad^4} \times \frac{ab^4c^5d^7}{1} \div \frac{a^6b^{10}c}{d^3}$$
$$= \frac{b^6c^6}{ad^4} \times \frac{ab^4c^5d^7}{1} \times \frac{d^3}{a^6b^{10}c}$$
$$= \frac{ab^{10}c^{11}d^{10}}{a^7b^{10}cd^4}$$
$$= \frac{c^{10}d^6}{a^6}$$

g) $\frac{(ef^{-1})^4}{g^{-2}h^3} \times \frac{e^6f^4g^2}{h} \times \left(\frac{e}{f^{-2}}\right)^{-1}$

$= \frac{e^4f^{-4}}{g^{-2}h^3} \times \frac{e^6f^4g^2}{h} \times \frac{f^{-2}}{e}$

$= \frac{e^4g^2}{f^4h^3} \times \frac{e^6f^4g^2}{h} \times \frac{1}{ef^2}$

$= \frac{e^{10}f^4g^4}{ef^6h^4}$

$= \frac{e^9g^4}{f^2h^4}$

h) $\left(\frac{1}{x} + \frac{x}{y}\right)^{-2}$

$= \left(\frac{y+x^2}{xy}\right)^{-2}$

$= \left(\frac{xy}{y+x^2}\right)^2$

$= \left(\frac{xy}{y+x^2}\right)\left(\frac{xy}{y+x^2}\right)$

$= \frac{x^2y^2}{y^2+2yx^2+x^4}$

i) $\left(\left(\frac{x}{y}\right)^{-2} + \left(\frac{x}{y} - \frac{y}{x}\right)^{-3}\right)^0$

$= 1$ (Anything to the power of Zero is one (1)).

j) $\frac{7^{x+1}\times 2^{2x-3}}{14^{x-5}}$

$= \frac{7^{x+1}\times 2^{2x-3}}{(7.2)^{x-5}}$

$= \frac{7^{x+1}2^{2x-3}}{7^{x-5}2^{x-5}}$

$= 7^{x+1-(x-5)}2^{2x-3-(x-5)}$

$= 7^{x+1-x+5}2^{2x-3-x+5}$

$= 7^6 2^{x+2}$

2. Solve for x:

a) $2^x - \frac{2^{x+1}}{5} = 0{,}3$

$\therefore 2^x - \frac{2^x2^1}{5} = \frac{3}{10}$

$\therefore 2^x\left(1 - \frac{2}{5}\right) = \frac{3}{10}$

$\therefore 2^x\left(\frac{3}{5}\right) = \frac{3}{10}$

$\therefore 2^x = \frac{1}{2}$

$\therefore 2^x = 2^{-1}$

$\therefore x = -1$

b) $2^{x+1} - \frac{2^{x+1}}{3} = 21\frac{1}{3}$

$\therefore 2^x2 - \frac{2^x2}{3} = 21\frac{1}{3}$

$\therefore 2^x\left(2 - \frac{2}{3}\right) = 21\frac{1}{3}$

$\therefore 2^x\left(\frac{4}{3}\right) = 21\frac{1}{3}$

$\therefore 2^x = 16$

$\therefore 2^x = 2^4$

$\therefore x = 4$

c) $10x^3 + 50 = 1\ 300$

$\therefore 10x^3 = 1250$

$\therefore x^3 = 125$

$\therefore x = \sqrt[3]{125}$

$\therefore x = 5$

d) $\frac{4x^4}{3} - 5 = 103$

$\therefore \frac{4x^4}{3} = 108$

$\therefore 4x^4 = 324$

$\therefore x^4 = 81$

$\therefore x = \sqrt[4]{81}$

$\therefore x = \pm 3$

e) $6^{2\frac{1}{2}} = x$

$\therefore x = 88{,}18$

f) $x = 4^{1\frac{1}{3}}$

$\therefore x = 6{,}35$

g) $4^{x+2} - 4^x = \frac{15}{16}$

$\therefore 4^x 4^2 - 4^x = \frac{15}{16}$

$\therefore 4^x(16-1) = \frac{15}{16}$

$\therefore 4^x(15) = \frac{15}{16}$

$\therefore 4^x = \frac{1}{16}$

$\therefore 4^x = 4^{-2}$

$\therefore x = -2$

h) $4^x - \frac{4^x}{2} = 1$

$\therefore 4^x\left(1 - \frac{1}{2}\right) = 1$

$\therefore 4^x\left(\frac{1}{2}\right) = 1$

$\therefore 4^x = 2$

$\therefore 2^{2x} = 2$

$\therefore 2x = 1$

$\therefore x = \frac{1}{2}$

i) $5^x + 3.5^x = 4$

$\therefore 5^x(1+3) = 4$

$\therefore 5^x(4) = 4$

$\therefore 5^x = 1$

$\therefore x = 0$

Anything to the power of zero is one.

j) $-\frac{1}{5}x^3 + \frac{3}{5} = -1$

$\therefore -\frac{1}{5}x^3 = -\frac{8}{5}$

$\therefore x^3 = 8$

$\therefore x = \sqrt[3]{8}$

$\therefore x = 2$

Activity 5

a) $(3x-5)(x^2+4x-6)$

$= 3x^3 + 12x^2 - 18x - 5x^2 - 20x + 30$

$= 3x^3 + 7x^2 - 38x + 30$

b) $\left(\frac{1}{2}y + 2\right)(4y^2 - 6y + 3)$

$= 2y^3 - 3y^2 + \frac{3}{2}y + 8y^2 - 12y + 6$

$= 2y^3 + 5y^2 - 10\frac{1}{2}y + 6$

c) $\left(\frac{1}{4}x - 1\right)\left(-x^2 - \frac{1}{3}x + 4\right)$

$= -\frac{1}{4}x^3 - \frac{1}{12}x^2 + x + x^2 + \frac{1}{3}x - 4$

$= -\frac{1}{4}x^3 + \frac{11}{12}x^2 + 1\frac{1}{3}x - 4$

d) $(6x+4y)(2x^2+4xy+3)$

$= 12x^3 + 24x^2y + 18x + 8x^2y + 16xy^2 + 12y$

$= 12x^3 + 32x^2y + 18x + 16xy^2 + 12y$

e) $(-4x+3)\left(\frac{1}{5}x^2+11x-6\right)$

$= -\frac{4}{5}x^3 - 44x^2 + 24x + \frac{3}{5}x^2 + 33x - 18$

$= -\frac{4}{5}x^3 - 43\frac{2}{5}x^2 + 57x - 18$

f) $(-3a-2b)(-7a^2+8ab-b^2)$

$= 21a^3 - 24a^2b + 3ab^2 + 14a^2b - 16ab^2 + 2b^3$

$= 21a^3 - 10a^2b - 13ab^2 + 2b^3$

g) $\left(\frac{3}{4}c^2-3d\right)\left(\frac{1}{2}c^2+cd+6d^2\right)$

$= \frac{3}{8}c^4 + \frac{3}{4}c^3d + 4\frac{1}{2}c^2\,d^2 - \frac{3}{2}c^2d - 3cd^2 - 18d^3$

h) $(5e+2f)\left(e^2-8ef+\frac{1}{3}f^2\right)$

$= 5e^3 - 40e^2f + \frac{5}{3}ef^2 + 2e^2f - 16ef^2 + \frac{2}{3}f^3$

$= 5e^3 - 38e^2f - 14\frac{1}{3}ef^2 + \frac{2}{3}f^3$

i) $(-2g+5h)(6g-6gh+h)$

$= -12g^2 + 12g^2h - 2gh + 30gh - 30gh^2 + 5h^2$

$= -12g^2 + 12g^2h + 28gh - 30gh^2 + 5h^2$

j) $\left(8k-\frac{4}{5}m\right)\left(2k^3+\frac{1}{4}k^2m-km\right)$

$= 16k^4 + 2k^3m - 8k^2m - 1\frac{3}{5}k^3m - \frac{1}{5}k^2m^2 + \frac{4}{5}km^2$

$= 16k^4 + \frac{2}{5}k^3m - 8k^2m - \frac{1}{5}k^2m^2 + \frac{4}{5}km^2$

Activity 6

1. Factorise the following by taking out a common factor:

a) $2a^2+4ab-8a^3b^2$

$2a(a+2b-4a^2b^2)$

b) $5c^6d^6 - 30c^4d^2 + 85c^5d^3$

$= 5c^4d^2(c^2d^4 - 6 + 17cd)$

To factorise with exponents look for variables that occur in every term and then take out the variable with the smallest exponent as the common factor.

c) $72ab^2c + 48bc^2 - 36ac$

$= 12c(6ab^2 + 4bc - 3a)$

d) $9xy + 3x^2y - 12x$

$= 3x(3y + xy - 4)$

2. Factorise the following cubes and squares:

a) $x^2 - y^2$

$= (x-y)(x+y)$

b) $2x^2 + 2y^2$

$= 2(x^2 + y^2)$

You can't factorise this any further as there is a plus between the two squares

c) $8x^3 - y^3$

$= (2x-y)(4x^2 + 2xy + y^2)$

d) $125 + 343g^3$

$= (5+7g)(25 - 35g + 49g^2)$

e) $54a - 16a^4$

$= 2a(27 - 8a^3)$

$= 2a(3-2a)(9 + 6a + 4a^2)$

f) $16c^2 - 25d^2$

$= (4c+5d)(4c-5d)$

g) $216b^3 + 8a^3$

$= 8(27b^3 + a^3)$

$= 8(3b+a)(9b^2 - 3ab + a^2)$

h) $x^3y - y^4$

$= y(x^3 - y^3)$

$= y(x-y)(x^2 + xy + y^2)$

i) $49e^3 - 64f^2e$

$= e(49e^2 - 64f^2)$

$= e(7e+8f)(7e-8f)$

j) $50x^4 - 800y^4$

$= 50(x^4 - 16y^4)$

$= 50(x^2 - 4y^2)(x^2 + 4y^2)$

$= 50\ (x-2y)(x+2y)(x^2 + 4y^2)$

3. Factorise the following trinomials:

a) $x^2 - x - 30$

$= (x+5)(x-6)$

b) $x^2 + 9x + 20$

$= (x+4)(x+5)$

c) $x^2 + 13x + 42$

$= (x+7)(x+6)$

d) $x^2 - 7x + 6$

$= (x-6)(x-1)$

e) $x^2 + x - 12$

$= (x+4)(x-3)$

f) $x^2 - 2\frac{1}{5}x + \frac{2}{5}$

$= (x-2)\left(x - \frac{1}{5}\right)$

g) $3x^2 - 7x - 6$

$= 3\left(x^2 - \frac{7}{3}x - 2\right)$

$= 3\left(x + \frac{2}{3}\right)(x-3)$

$= (3x+2)(x-3)$

h) $2x^2 - 11x + 5$

$= 2\left(x^2 - \frac{11}{2}x + \frac{5}{2}\right)$

$= 2\left(x - \frac{1}{2}\right)(x-5)$

$= (2x-1)(x-5)$

i) $4x^2 - 19x - 5$

$= 4\left(x^2 - \frac{19}{4}x - \frac{5}{4}\right)$

$= 4\left(x + \frac{1}{4}\right)(x - 5)$

$= (4x + 1)(x - 5)$

j) $6x^2 + 7x - 5$

$= 6\left(x^2 + \frac{7}{6}x - \frac{5}{6}\right)$

$= 6\left(x - \frac{1}{2}\right)\left(x + \frac{5}{3}\right)$

$= 2\left(x - \frac{1}{2}\right).3\left(x + \frac{5}{3}\right)$

$= (2x - 1)(3x + 5)$

Because there are two fractions and 6 has the factors 2 and 3 we "split" the 6 up to go into both brackets so that we can "get rid" of both fractions.

k) $3x^2 - 14x + 8$

$= 3\left(x^2 - \frac{14}{3} + \frac{8}{3}\right)$

$= 3\left(x - \frac{2}{3}\right)(x - 4)$

$= (3x - 2)(x - 4)$

l) $3x^2 - 5x + 2$

$= 3\left(x^2 - \frac{5}{3}x + \frac{2}{3}\right)$

$= 3\left(x - \frac{2}{3}\right)(x - 1)$

$= (3x - 2)(x - 1)$

m) $2x^2 - 11x - 21$

$= 2\left(x^2 - \frac{11}{5}x - \frac{21}{2}\right)$

$= 2\left(x + \frac{3}{2}\right)(x - 7)$

$= (2x + 3)(x - 7)$

n) $2x^2 - 27x + 81$

$= 2\left(x^2 - \frac{27}{2}x + \frac{81}{2}\right)$

$= 2\left(x - \frac{9}{2}\right)(x - 9)$

$= (2x - 9)(x - 9)$

o) $2x^2 + 3x - 5$

$= 2\left(x^2 + \frac{3}{2} - \frac{5}{2}\right)$

$= 2\left(x + \frac{5}{2}\right)(x - 1)$

$= (2x + 5)(x - 1)$

p) $4x^2 + 20x + 25$

$= 4\left(x^2 + 5x + \frac{25}{4}\right)$

$= 4\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right)$

$= (2x + 5)(2x + 5)$

$= (2x + 5)^2$

"Split" the 4 into 2 × 2

4. Factorise the following by grouping:

a) $8ab - 12a^2b - 20b^2 + 30ab^2$

$= 2b(4a - 6a^2 - 10b + 15ab)$

$= 2b\big(2a(2 - 3a) + 5b(-2 + 3a)\big)$

$= 2b\big(2a(2 - 3a) - 5b(2 - 3a)\big)$

$= 2b(2 - 3a)(2a - 5b)$

b) $c^3 + 18d^2 - 6cd - 3c^2d$

$= c^3 - 3c^2d + 18d^2 - 6cd$

$= c^2(c - 3d) + 6d(3d - c)$

$= c^2(c - 3d) - 6d(c - 3d)$

$= (c - 3d)(c^2 - 6d)$

To make the brackets the same take a common factor of -1 from the bracket and multiply by the common factor in the front of the bracket.

c) $-12ab^2 - 18a^4b^2 + 20a^2b + 30a^5b$

$= 2ab(-6b - 9a^3b + 10a + 15a^4)$

$= 2ab\big(-3b(2 + 3a^3) + 5a(2 + 3a^3)\big)$

$= 2ab(2 + 3a^3)(5a - 3b)$

d) $2e^2f^3 - 6e^2f - 3ef^3 + 9ef$

$= 2e^2f^3 - 3ef^3 + 9ef - 6e^2f$

$= ef^3(2e - 3) + 3ef(3 - 2e)$

$= ef^3(2e - 3) - 3ef(2e - 3)$

$= (2e - 3)(ef^3 - 3ef)$

$= ef(2e - 3)(f^2 - 3)$

e) $16g^3 - 64g^2h - gh + 4h^2$
$= 16g^2(g - 4h) - h(g - 4h)$
$= (g - 4h)(16g^2 - h)$

f) $2j^3 - 12k^3 + 3kj - 8j^2k^2$
$= 2j^3 + 3kj - 12k^3 - 8j^2k^2$
$= j^2(2j^2 + 3k) - 4k^2(3k + 2j^2)$
$= (2j^2 + 3k)(j^2 - 4k^2)$

g) $3m^3 - 14n^4 + 21mn - 2m^2n^3$
$= 3m^3 - 2m^2n^3 + 21mn - 14n^4$
$= m^2(3m - 2n^3) + 7n(3m - 2n^3)$
$= (3m - 2n^3)(m^2 + 7n)$

h) $q^3 - 3p^2q - 8q^2p + 24p^3$
$= q^3 - 8q^2p + 24p^3 - 3p^2q$
$= q^2(q - 8p) + 3p^2(8p - q)$
$= q^2(q - 8p) - 3p^2(q - 8p)$
$= (q - 8p)(q^2 - 3p^2)$

i) $7r^3 - 56r^2t + 10rt - 80t^2$
$= 7r^2(r - 8t) + 10t(r - 8t)$
$= (r - 8t)(7r^2 + 10t)$

j) $v - 15v^2w^2 + 5v^3w - 3w$
$= v - 3w + 5v^3w - 15v^2w^2$
$= 1(v - 3w) + 5v^2w(v - 3w)$
$= (v - 3w)(1 + 5v^2w)$

5. Factorise the following: (use any of the above methods)

a) $2x^2 - 49x - 25$
$= 2\left(x^2 - \frac{49}{2}x - \frac{25}{2}\right)$
$= 2\left(x + \frac{1}{2}\right)(x - 25)$
$= (2x + 1)(x - 25)$

b) $x^2 - 6x + 9$
$= (x - 3)(x - 3)$
$= (x - 3)^2$

c) $80x^3 - 500xy^2$
$= 20x(4x^2 - 25y^2)$
$= 20x(2x - 5y)(2x + 5y)$

d) $6x^3 - 48y^3$
$= 6(x^3 - 8y^3)$
$= 6(x - 2y)(x^2 + 2xy + 4y^2)$

e) $13a^3 - 30b^3 - 78ab + 5a^2b^2$
$= 13a^3 - 78ab + 5a^2b^2 - 30b^3$
$= 13a(a^2 - 6b) + 5b^2(a^2 - 6b)$
$= (a^2 - 6b)(13a + 5b^2)$

f) $4x^2 - 11x + 6$
$= 4\left(x^2 - \frac{11}{4}x + \frac{6}{4}\right)$
$= 4\left(x - \frac{3}{4}\right)(x - 2)$
$= (4x - 3)(x - 2)$

g) $3x^2 + 6x - 24$
$= 3(x^2 + 2x - 8)$
$= 3(x + 4)(x - 2)$

h) $\frac{1}{25}x^2 - 16y^4$
$= \left(\frac{1}{5}x - 4y^2\right)\left(\frac{1}{5}x + 4y\right)$

i) $6x^2 + 5x - 4$

$= 6\left(x^2 + \frac{5}{6}x - \frac{4}{6}\right)$

$= 6\left(x - \frac{1}{2}\right)\left(x + \frac{4}{3}\right)$

$= (2x - 1)(3x + 4)$

j) $a^3b^6 + 125c^3$

$= (ab^2 + 5c)(a^2b^4 - 5ab^2c + 25c^2)$

k) $10x^2 - 3x - 18$

$= 10\left(x^2 - \frac{3}{10}x - \frac{18}{10}\right)$

$= 10\left(x + \frac{6}{5}\right)\left(x - \frac{3}{2}\right)$

$= (5x + 6)(2x - 3)$

l) $\frac{1}{3}x^2 - 1\frac{2}{3}x + 2$

$= \frac{1}{3}(x^2 - 5x + 6)$

$= \frac{1}{3}(x - 3)(x - 2)$

$= \left(\frac{1}{3}x - 1\right)(x - 2)$

m) $8a^3 - 42b^3 - 6a^2b^2 + 56ab$

$= 8a^3 + 56ab - 42b^3 - 6a^2b^2$

$= 8a(a^2 + 7b) - 6b^2(7b + a^2)$

$= (a^2 + 7b)(8a - 6b^2)$

$= 2(a^2 + 7b)(4a - 3b^2)$

n) $0{,}01x^2 + y^2$

= can't factorise – not a difference of squares.

o) $2x^2 - \frac{1}{2}x - \frac{1}{4}$

$= 2\left(x^2 - \frac{1}{4}x - \frac{1}{8}\right)$

$= 2\left(x - \frac{1}{2}\right)\left(x + \frac{1}{4}\right)$

$= (2x - 1)\left(x + \frac{1}{4}\right)$

p) $6x^2 + 43x + 65$

$= 6\left(x^2 + \frac{43}{6}x + \frac{65}{6}\right)$

$= 6\left(x + \frac{13}{6}\right)(x + 5)$

$= (6x + 13)(x + 5)$

q) $2x^2 - 9x - 110$

$= 2\left(x^2 - \frac{9}{2}x - 55\right)$

$= 2\left(x + \frac{11}{2}\right)(x - 10)$

$= (2x + 11)(x - 10)$

r) $-81x^4 - 24xy^3$

$= -3x(27x^3 + 8y^3)$

$= -3x(3x + 2y)(9x^2 - 6xy + 4y^2)$

s) $a^3 - 7ab + 112b^3 - 16a^2b^2$

$= a(a^2 - 7b) + 16b^2(7b - a^2)$

$= a(a^2 - 7b) - 16b^2(a^2 - 7b)$

$= (a^2 - 7b)(a - 16b^2)$

t) $x^4 - 18x^2 + 32$

$= (x^2 - 2)(x^2 - 16)$

$= (x^2 - 2)(x - 4)(x + 4)$

u) $3x^2 + 7x - 40$

$= 3\left(x^2 + \frac{7}{3}x - \frac{40}{3}\right)$

$= 3\left(x + \frac{8}{3}\right)(x-5)$

$= (3x+8)(x-5)$

v) $4x^2 - 5x - 6$

$= 4\left(x^2 - \frac{5}{4}x - \frac{6}{4}\right)$

$= 4\left(x + \frac{3}{4}\right)(x-2)$

$= (4x+3)(x-2)$

w) $16x^4 - y^4$

$= (4x^2 - y^2)(4x^2 + y^2)$

$= (2x-y)(2x+y)(4x^2+y^2)$

x) $x^6 + y^6$

$= (x^2+y^2)(x^4 - x^2y^2 + y^4)$

Activity 7

a) $\frac{5x^2-20y^2}{5xy} \times \frac{2xy^2}{5x+10y}$

$= \frac{5(x^2-4y^2)}{5xy} \times \frac{2xy^2}{5(x+2y)}$

$= \frac{(x-2y)(x+2y)}{1} \times \frac{2y}{5(x+2y)}$

$= \frac{2y(x-2y)}{5}$

b) $\frac{a}{a-b} - \frac{b}{b-a}$

$= \frac{a}{a-b} + \frac{b}{a-b}$

$= \frac{a+b}{a-b}$

Remember that you can take out a common factor of -1 to make the denominators the same ☺ Your negative in front of the fraction becomes positive.

c) $\frac{p}{3p-6q} + \frac{2p-4q}{p^2-4q^2}$

$= \frac{p}{3(p-2q)} + \frac{2\cancel{(p-2q)}}{\cancel{(p-2q)}(p+2q)}$

$= \frac{p}{3(p-2q)} + \frac{2}{p+2q}$

$= \frac{p(p+2q)+2(3)(p-2q)}{3(p+2q)(p-2q)}$

$= \frac{p^2+2pq+6p-12q}{3(p+2q)(p-2q)}$

d) $\frac{x^3-8y^3}{x^2-4y^2} \div \frac{x^2+2xy+4y^2}{6x+12y}$

$= \frac{\cancel{(x-2y)}(x^2+2xy+4y^2)}{\cancel{(x-2y)}(x+2y)} \div \frac{(x^2+2xy+4y^2)}{6(x+2y)}$

$= \frac{\cancel{(x^2+2xy+4y^2)}}{\cancel{(x+2y)}} \times \frac{6\cancel{(x+2y)}}{\cancel{(x^2+2xy+4y^2)}}$

$= 6$

e) $\frac{m+2n}{2mn} + \frac{m}{2m-4n} - \frac{3m+1}{2n-m}$

$= \frac{m+2n}{2mn} + \frac{m}{2(m-2n)} + \frac{3m+1}{(m-2n)}$

$= \frac{(m+2n)(m-2n)+m(mn)+(3m+1)(2mn)}{2mn(m-2n)}$

$= \frac{m^2-4n^2+m^2n+6m^2n+2mn}{2mn(m-2n)}$

$= \frac{m^2-4n^2+7m^2n+2mn}{2mn(m-2n)}$

f) $\frac{x^2-4}{x^3-8} \times \frac{2-x}{x+2}$

$= \frac{\cancel{(x-2)}\cancel{(x+2)}}{\cancel{(x-2)}(x^2+2x+4)} \times \frac{2-x}{\cancel{x+2}}$

$= \frac{2-x}{x^2+2x+4}$

g) $\dfrac{a+b}{a^2-b^2}\times\dfrac{6a^2b}{3a+6b}\div\dfrac{2ab}{a^3-b^3}$

$$=\frac{\cancel{(a+b)}}{\cancel{(a-b)}\cancel{(a+b)}}\times\frac{\cancel{6a^2b}}{\cancel{3}(a+2b)}\times\frac{\cancel{(a-b)}\left(a^2+ab+b^2\right)}{\cancel{2ab}}$$
$$=\frac{a\left(a^2+ab+b^2\right)}{a+2b}$$

h) $\dfrac{a}{a+b}+\dfrac{1}{b-a}\times\dfrac{a^2-b^2}{2a-3b}$

$$=\frac{a}{a+b}-\frac{1}{\cancel{a-b}}\times\frac{\cancel{(a-b)}(a+b)}{2a-3b}$$
$$=\frac{a}{a+b}-\frac{(a+b)}{2a-3b}$$
$$=\frac{a(2a-3b)-(a+b)(a+b)}{(a+b)(2a-3b)}$$
$$=\frac{2a^2-3ab-\left(a^2+2ab+b^2\right)}{(a+b)(2a-3b)}$$
$$=\frac{2a^2-3ab-a^2-2ab-b^2}{(a+b)(2a-3b)}$$
$$=\frac{a^2-5ab-b^2}{(a+b)(2a-3b)}$$

i) $\dfrac{x^2y}{3x^3+81y^3}\div\dfrac{x+2y}{3x^4-48y^4}\times\dfrac{x+3y}{x-2y}$

$$=\frac{x^2y}{3(x^3+27y^3)}\div\frac{x+2y}{3(x^4-16y^4)}\times\frac{x+3y}{x-2y}$$
$$=\frac{x^2y}{\cancel{3}\cancel{(x+3y)}(x^2-3xy+9y^2)}\times\frac{\cancel{3}\left(x^2-4y^2\right)\left(x^2+4y^2\right)}{x+2y}\times\frac{\cancel{(x+3y)}}{(x-2y)}$$
$$=\frac{x^2y}{(x^2-3xy+9y^2)}\times\frac{\cancel{(x-2y)}\cancel{(x+2y)}\left(x^2+4y^2\right)}{\cancel{(x+2y)}}\times\frac{1}{\cancel{(x-2y)}}$$
$$=\frac{x^2y\left(x^2+4y^2\right)}{x^2-3xy+9y^2}$$

j) $\dfrac{3}{a+b}-\dfrac{2}{a^2-b^2}+\dfrac{5}{b-a}$

$$=\frac{3}{a+b}-\frac{2}{(a-b)(a+b)}-\frac{5}{a-b}$$
$$=\frac{3(a-b)-2-5(a+b)}{(a+b)(a-b)}$$
$$=\frac{3a-3b-2-5a-5b}{(a+b)(a-b)}$$
$$=\frac{-2a-8b-2}{a^2-b^2}$$
$$=\frac{-2(a+4b+1)}{(a+b)(a-b)}$$

k) $\dfrac{-2(x+y)}{x^3-y^3}+\dfrac{1}{x-y}-\dfrac{6}{x^2+xy+y^2}$

$$=\frac{-2(x+y)}{(x-y)(x^2+xy+y^2)}+\frac{1}{x-y}-\frac{6}{x^2+xy+y^2}$$
$$=\frac{-2(x+y)+1\left(x^2+xy+y^2\right)-6\,(x-y)}{(x-y)(x^2+xy+y^2)}$$
$$=\frac{-2x-2y+x^2+xy+y^2-6x+6y}{(x-y)(x^2+xy+y^2)}$$
$$=\frac{x^2-8x+xy+4y+y^2}{(x-y)(x^2+xy+y^2)}$$

l) $\dfrac{8x^3+125}{x^2+4x+4}\times\dfrac{-2-x}{x+5}\times\dfrac{4x+8}{2x+5}$

$$=\frac{\cancel{(2x+5)}\left(4x^2-10x+25\right)}{\cancel{(x+2)}\cancel{(x+2)}}\times\frac{-\cancel{(x+2)}}{(x+5)}\times\frac{4\cancel{(x+2)}}{\cancel{2x+5}}$$
$$=\frac{-4\left(4x^2-10x+25\right)}{(x+5)}$$

m) $\dfrac{v^2+vw}{v^2-w^2}\times\dfrac{w-v}{2v+4w}\div\dfrac{5v+8w}{4w+2v}$

$$=\frac{v(v+w)}{(v-w)(v+w)}\times\frac{-(v-w)}{2(v+2w)}\times\frac{2(v+2w)}{5v+8w}$$
$$=\frac{-v}{5v+8w}$$

n) $\dfrac{1}{a+b}+\dfrac{3}{a-b}-\dfrac{2a+b}{b^2-a^2}$

$$=\frac{1}{a+b}+\frac{3}{a-b}+\frac{2a+b}{a^2-b^2}$$
$$=\frac{1}{a+b}+\frac{3}{a-b}+\frac{2a+b}{(a-b)(a+b)}$$
$$=\frac{1(a-b)+3(a+b)+2a+b}{(a-b)(a+b)}$$
$$=\frac{a-b+3a+3b+2a+b}{a^2-b^2}$$
$$=\frac{6a+3b}{a^2-b^2}$$
$$=\frac{3(2a+b)}{(a-b)(a+b)}$$

Activity 8

1. a) $2x - 1 = -9x + 6$

$2x + 9x = 6 + 1$

$11x = 7x$

$x = \frac{7}{11}$

b) $6x - 1 = 3x - 2$

$6x - 3x = -2 + 1$

$3x = -1$

$x = -\frac{1}{3}$

c) $\frac{1}{2}x - 10 = 4x + 4$

$\frac{1}{2}x - 4x = 4 + 10$

$-3\frac{1}{2}x = 14$

$x = -4$

d) $-5x + 1 = 4x - 8$

$-5x - 4x = -8 - 1$

$-9x = -9$

$x = 1$

e) $3x - 5 = x + 13$

$3x - x = 13 + 5$

$2x = 18$

$x = 9$

f) $-2x - 5 = 7x + 2$

$-2x - 7x = 2 + 5$

$-9x = 7$

$x = -\frac{7}{9}$

g) $-\frac{1}{2}x - \frac{1}{6} = 5x + 1\frac{5}{6}$

$-\frac{1}{2}x - 5x = 1\frac{5}{6} + \frac{1}{6}$

$-\frac{11}{2}x = 2$

$x = -\frac{4}{11}$

h) $3(x + 5) = 3x + 9$

$3x + 15 = 3x + 9$

$3x - 3x = 9 - 15$

$0 = -4$

$\therefore x$ does not exist

i) $x - 2 = 4(2x + 3)$

$x - 2 = 8x + 12$

$x - 8x = 12 + 2$

$-7x = 14$

$x = -2$

j) $x + 7 = 3(2x + 4)$

$x + 7 = 6x + 12$

$x - 6x = 12 - 7$

$-5x = 5$

$x = -1$

2. a) $x^2 + 7x + 10 = 0$

$(x + 5)(x + 2) = 0$

$\therefore x + 5 = 0 \quad OR \quad x + 2 = 0$

$\therefore x = -5 \qquad x = -2$

b) $x^2 + 12x + 36 = 0$

$(x + 6)(x + 6) = 0$

$\therefore x + 6 = 0$

$\therefore x = -6$

Only have to do this once because the factors are the same so it will give you the same answer.

c) $x^2 + 7x - 44 = 0$

$(x + 11)(x - 4) = 0$

$\therefore x + 11 = 0 \quad OR \quad x - 4 = 0$

$\therefore x = -11 \qquad x = 4$

d) $x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$\therefore x - 4 = 0 \quad OR \quad x - 1 = 0$

$\therefore x = 4 \qquad x = 1$

e) $2x^2 - 13x + 15 = 0$

$2\left(x^2 - \frac{13}{2}x + \frac{15}{2}\right) = 0$

$2\left(x - \frac{3}{2}\right)(x - 5) = 0$

$(2x - 3)(x - 5) = 0$

$\therefore 2x - 3 = 0 \quad OR \quad x - 5 = 0$

$\therefore 2x = 3 \qquad x = 5$

$\therefore x = \frac{3}{2}$

f) $x^2 - 16 = 0$

$(x - 4)(x + 4) = 0$

$\therefore x - 4 = 0 \quad OR \quad x + 4 = 0$

$\therefore x = 4 \qquad x = -4$

g) $2x^2 - 8 = 0$

$2(x^2 - 4) = 0$

$(x - 2)(x + 2) = 0$

Divide both sides by 2 Remember zero divided by anything is zero.

$\therefore x - 2 = 0 \quad OR \quad x + 2 = 0$

$\therefore x = 2 \qquad x = -2$

h) $3x^2 - 5x + 2 = 0$

$3\left(x^2 - \frac{5}{3}x + \frac{2}{3}\right) = 0$

$3\left(x - \frac{2}{3}\right)(x - 1) = 0$

$(3x - 2)(x - 1) = 0$

$\therefore 3x - 2 = 0 \quad OR \quad x - 1 = 0$

$\therefore 3x = 2 \qquad x = 1$

$\therefore x = \frac{2}{3}$

i) $5x^2 - 42x - 27 = 0$

$5\left(x^2 - \frac{42}{5}x - \frac{27}{5}\right) = 0$

$5\left(x + \frac{3}{5}\right)(x - 9) = 0$

$(5x + 3)(x - 9) = 0$

$\therefore 5x + 3 = 0 \quad OR \quad x - 9 = 0$

$\therefore 5x = -3 \qquad x = 9$

$\therefore x = -\frac{3}{5}$

j) $3x^2 = 16 - 8x$

$3x^2 + 8x - 16 = 0$

$3\left(x^2 + \frac{8}{3}x - \frac{16}{3}\right) = 0$

$3\left(x - \frac{4}{3}\right)(x + 4) = 0$

$(3x - 4)(x + 4) = 0$

$\therefore 3x - 4 = 0 \quad OR \quad x + 4 = 0$

$\therefore 3x = 4 \qquad x = -4$

$\therefore x = \frac{4}{3}$

Note: in question 2(g) we took out the common factor or 2 and then "got rid" of it by dividing both sides by 2. In a trinomial where there is a constant in front of the x^2 we can also divide by that common factor and leave the factors in the bracket as a fraction. This will actually save you time in the exam.
Check out your factors in (i) and (j) above and your final answers for x – do you see that they are the same?

3. a) $y = 2x - 3$ ..1 and $2y = 5x - 8$ ...2

Substitute 1 into 2

$\therefore 2(2x - 3) = 5x - 8$

$\therefore 4x - 6 = 5x - 8$

$\therefore 4x - 5x = -8 + 6$

$\therefore -x = -2$

$\therefore x = 2$

Substitute the answer back into 1

$\therefore y = 2(2) - 3$

$\therefore y = 1$

$\therefore (2; 1)$

b) $3y = -4x + 1$ ..1 and $2y = x - 4$

$y = \frac{1}{2}x - 2$ ...2

Substitute 2 into 1

$\therefore 3\left(\frac{1}{2}x - 2\right) = -4x + 1$

$\therefore \frac{3}{2}x - 6 = -4x + 1$

$\therefore \frac{3}{2}x + 4x = 1 + 6$

$\therefore \frac{11}{2}\ x = 7$

$\therefore x = \frac{14}{11}$

Substitute back into 2

$\therefore y = \frac{1}{2}\left(\frac{14}{11}\right) - 2$

$\therefore y = -\frac{15}{11}$

$\therefore \left(\frac{14}{11}; -\frac{15}{11}\right)$

c) $3y = x$ ..1 and $y = 2x + 5$ ...2

Substitute 1 into 2

$\therefore y = 2(3y) + 5$

$\therefore y = 6y + 5$

$\therefore y - 6y = 5$

$\therefore -5y = 5$

$\therefore y = -1$

Substitute back into 1

$\therefore x = 3(-1)$

$\therefore x = -3$

$\therefore (-3; -1)$

d) $y + x = 1$ and $5y = -x + 3$ ...2

$y = 1 - x$ ..1

Substitute 1 into 2

$\therefore 5(1 - x) = -x + 3$

$\therefore 5 - 5x = -x + 3$

$\therefore -5x + x = 3 - 5$

$\therefore -4x = -2$

$\therefore x = \frac{1}{2}$

Substitute back into 1

$\therefore y = 1 - \frac{1}{2}$

$\therefore y = \frac{1}{2}$

$\therefore \left(\frac{1}{2}; \frac{1}{2}\right)$

e) $-2y = 5x + 6$ ..1 and $y = 4x - 5$ ..2

Substitute 2 into 1

$\therefore\ -2(4x - 5) = 5x + 6$

$\therefore\ -8x + 10 = 5x + 6$

$\therefore\ -8x - 5x = 6 - 10$

$\therefore\ -13x = -4$

$\therefore x = \frac{4}{13}$

Substitute back into 2

$\therefore y = 4\left(\frac{4}{13}\right) - 5$

$\therefore y = -3\frac{10}{13}$

$\therefore \left(\frac{4}{13};\ -3\frac{10}{13}\right)$

f) $y = -5x$ ..1 and $2y = -2x - 1$ ..2

Substitute 1 into 2

$\therefore 2(-5x) = -2x - 1$

$\therefore\ -10x = -2x - 1$

$\therefore\ -10x + 2x = -1$

$\therefore\ -8x = -1$

$\therefore x = \frac{1}{8}$

Substitute back into 1

$\therefore y = -5\left(\frac{1}{8}\right)$

$\therefore y = -\frac{5}{8}$

$\therefore \left(\frac{1}{8};\ -\frac{5}{8}\right)$

g) $5y = 2x + 8$ ..1 and $2y = x + 4$

Substitute 2 into 1 $\qquad y = \frac{1}{2}x + 2$ ...2

$\therefore 5\left(\frac{1}{2}x + 2\right) = 2x + 8$

$\therefore \frac{5}{2}x + 10 = 2x + 8$

$\therefore \frac{5}{2}x - 2x = 8 - 10$

$\therefore \frac{1}{2}x = -2$

$\therefore\ x = -4$

Substitute back into 1

$\therefore y = \frac{1}{2}(-4) + 2$

$\therefore y = 0$

∴ (-4; 0)

h) $y = 4x + 5$ ..1 and $3x - y = 2$ ...2

Substitute 1 into 2

$\therefore 3x - (4x + 5) = 2$

$\therefore 3x - 4x - 5 = 2$

$\therefore\ -x = 2 + 5$

$\therefore -x = 7$

$\therefore x = \ -7$

Substitute back into 1

$\therefore y = 4(-7) + 5$

$\therefore y = \ -23$

∴ (-7; -23)

i) $y = 2x - 4$ ...1 and $x - 8y = 27$ ...2

Substitute 1 into 2

$\therefore x - 8(2x - 4) = 27$

$\therefore x - 16x\ + 32 = 27$

$\therefore\ -15x = 27 - 32$

$\therefore\ -15x = \ -5$

$\therefore x = \frac{1}{3}$

Substitute back into 1

$\therefore y = 2\left(\frac{1}{3}\right) - 4$

$\therefore y = \ -3\frac{1}{3}$

$\therefore \left(\frac{1}{3};\ -3\frac{1}{3}\right)$

j) $6y - x = 0$ ...1 and $y = \ -5 + \frac{1}{3}x$ ...2

Substitute 2 into 1

$\therefore 6\left(-5 + \frac{1}{3}x\right) - x = 0$

$\therefore\ -30 + 2x - x = 0$

$\therefore x = 30$

Substitute back into 2

$\therefore y = \ -5 + \frac{1}{3}(30)$

$\therefore y = 5$

∴ (30; 5)

4. a)

	Now	In 5 years' time
Sarah	$x - 20$	$x - 20 + 5$
Dad	x	$x + 5$

$\therefore x - 20 + 5 = \frac{1}{2}(x + 5)$

$\therefore x - 15 = \frac{1}{2}x + \frac{5}{2}$

$\therefore x - \frac{1}{2}x = \frac{5}{2} + 15$

$\therefore \frac{1}{2}x = 17\frac{1}{2}$

$\therefore x = 35$

Sarah is now 35 − 20 = 15 years

Dad is 35

b)

	Speed	Distance	Time
There	$\frac{10}{2x}$	10km	$2x$
Back	$\frac{10}{x}$	10km	x

$\frac{1}{2}\left(\frac{10}{2x}+\frac{10}{x}\right)=25$

$\therefore \frac{1}{2}\left(\frac{5}{x}+\frac{10}{x}\right)=25$

$\therefore \frac{15}{2x}=25$

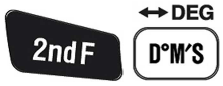
$\therefore 15=50x$

$\therefore \frac{15}{50}=x$

$\therefore x=\frac{3}{10}$ Or 18 minutes from school

And 36 minutes to school

In order to find the time once you have got your answer $\frac{3}{5}$ simply press



and then the calculator will give it back to you in the format 0 ° 18 ' 0."
which means 0 hours, 18 minutes and 0 seconds.
Easy peasy ☺

c) Let coke $=x$ and chips $=y$

$\therefore 3x+y=27$ and $2x+3y=25$ 2

$\therefore y=27-3x$ 1

Substitute 2 into 1

$\therefore 2x+3(27-3x)=25$

$\therefore 2x+81-9x=25$

$\therefore -7x=25-81$

$\therefore -7x=-56$

$\therefore x=8$

Substitute back into 1

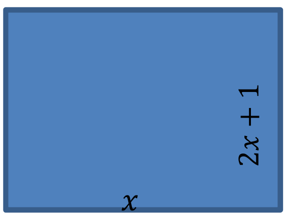
$\therefore y=27-3(8)$

$\therefore y=3$

$\therefore$ A coke costs R8 and a packet of chips costs R3.

Don't forget to end your answer by writing a sentence that shows that you answered the question and understood the question.

d)



$\therefore x(2x+1)=300$

$\therefore 2x^2+x-300=0$

$\therefore 2\left(x^2+\frac{1}{2}x-150\right)=0$

$\therefore 2\left(x+\frac{25}{2}\right)(x-12)=0$

$\therefore (2x+25)(x-12)=0$

$\therefore 2x+25=0 \quad Or \quad x-12=0$

$\therefore x=-\frac{25}{2} \qquad x=12$

Length cannot be negative so $x=12$

$\therefore P = 2l + 2b$
$\therefore P = 2(12) + 2(2(12) + 1)$
$\therefore P = 24 + 50$
$\therefore P = 74km.$ ∴Dan needs 74km of fencing for his farm.

e) $P = 100 = 2b + 2l$

$\therefore 100 = 2(x + 5) + 2l$
$\therefore 100 - 2x - 10 = 2l$
$\therefore 90 - 2x = 2l$
$\therefore l = 45 - x$

$\therefore Area = 400 = l \times b$
$\therefore 400 = (45 - x)(x + 5)$
$\therefore 400 = 45x + 225 - x^2 - 5x$
$\therefore 400 = 40x + 225 - x^2$
$\therefore 0 = -x^2 + 40x + 225 - 400$
$\therefore 0 = x^2 - 40x + 175$

$\therefore x - 5 = 0 \quad OR \quad x - 35 = 0$
$\therefore x = 5 \qquad x = 35$

∴ the length is = 45 -5 = 40m OR the length is = 45 -35 = 10m

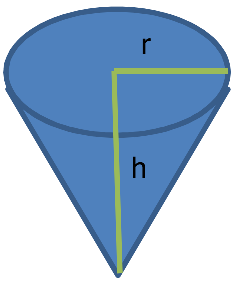
f) $SA = 33 = (x + 3)(x - 5)$

$\therefore 33 = x^2 - 5x + 3x - 15$
$\therefore 0 = x^2 - 2x - 15 - 33$
$\therefore 0 = x^2 - 2x - 48$
$\therefore 0 = (x + 6)(x - 8)$

$\therefore x + 6 = 0 \quad OR \quad x - 8 = 0$
$\therefore x = -6 \qquad x = 8$

Length cannot be negative so x =8.

g)



$v = 47{,}1239 = \frac{1}{3}\pi r^2 h$
$\therefore 47{,}1239 = \frac{1}{3}\pi r^2 (5)$
$\therefore 141{,}3717 = 5\pi r^2$
$\therefore 141{,}3717 \div 5\pi = r^2$
$\therefore 9{,}0000 \ldots = r^2$
$\therefore r = \pm 3$
$\therefore r = 3$

∴Anne's ice-cream scoop can be 6cm wide.

5. a) Find r if $v = \pi r^2$

$\therefore \frac{v}{\pi} = \frac{\pi r^2}{\pi}$

$\therefore r^2 = \frac{v}{\pi}$

$\therefore r = \pm\sqrt{\frac{v}{\pi}}$

b) Find t if $s = \frac{d}{t}$

$\therefore st = d$

$\therefore \frac{st}{s} = \frac{d}{s}$

$\therefore t = \frac{d}{s}$

c) Find *i* if $A = P(1+i)^n$

$\therefore \frac{A}{P} = (1+i)^n$

$\therefore \sqrt[n]{\frac{A}{P}} = 1 + i$

$\therefore -1 + \sqrt[n]{\frac{A}{P}} = i$

d) Find r if $v = \frac{4}{3}\pi r^3$

$\therefore \frac{3}{4}v = \pi r^3$

$\therefore \frac{3v}{4\pi} = r^3$

$\therefore r = \sqrt[3]{\frac{3v}{4\pi}}$

e) Find h if $v = \pi r^2 h$

$\therefore \frac{v}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$\therefore h = \frac{v}{\pi r^2}$

f) Find m if $y = mx + c$

$\therefore y - c = mx$

$\therefore x = \frac{y-c}{m}$

g) Find d if $T = a + (n-1)d$

$\therefore T - a = (n-1)d$

$\therefore \frac{T-a}{n-1} = d$

h) Find x if $y = -ax^2 + 3$

$\therefore y - 3 = -ax^2$

$\therefore \frac{y-3}{-a} = x^2$

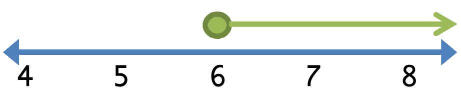
$\therefore x = \pm\sqrt{\frac{y-3}{-a}}$

6. a) $3x + 4 \leq 5x - 8$

$3x - 5x \leq -8 - 4$

$-2x \leq -12$

$x \geq 6$



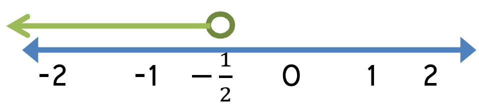
$x \in [6;\ \infty)$

b) $-2x + 5 > 6$

$-2x > 6 - 5$

$-2x > 1$

$x < -\frac{1}{2}$



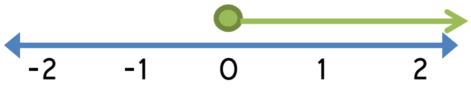
$x \in \left(-\infty;\ -\frac{1}{2}\right)$

c) $2x + 5 \geq 5$

$2x \geq 5 - 5$

$2x \geq 0$

$x \geq 0$

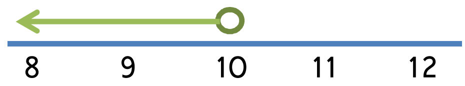


$x \in [0;\ \infty)$

d) $x - 6 < 4$

$x < 4 + 6$

$x < 10$



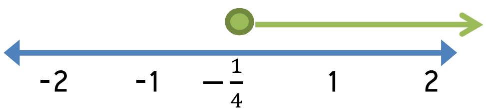
$x \in (-\infty; 10)$

e) $-4x + 1 \leq 2$

$-4x \leq 2 - 1$

$-4x \leq 1$

$x \geq -\frac{1}{4}$



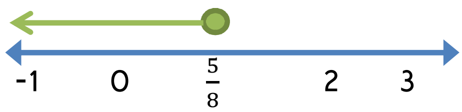
$x \in \left[-\frac{1}{4}\ ;\ \infty\right)$

f) $5 - 6x \geq 2x$

$-6x - 2x \geq -5$

$-8x \geq -5$

$x \leq \frac{5}{8}$



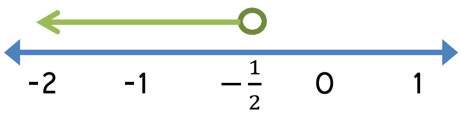
$x \in \left(-\infty; \frac{5}{8}\right]$

g) $x - 6 > 9x - 2$

$x - 9x > -2 + 6$

$-8x > 4$

$x < -\frac{1}{2}$



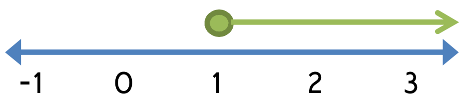
$x \in \left(-\infty;\ -\frac{1}{2}\right)$

h) $-x + 3 \leq 2$

$-x \leq 2 - 3$

$-x \leq -1$

$x \geq 1$



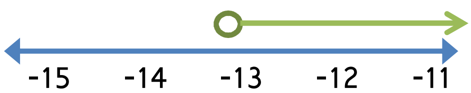
$x \in [1;\ \infty)$

i) $x - 8 < 2x + 5$

$x - 2x < 5 + 8$

$-x < 13$

$x > -13$



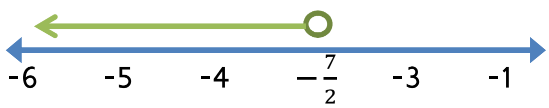
$x \in (-13;\ \infty)$

j) $-2x + 3 > 10$

$-2x > 10 - 3$

$-2x > 7$

$x < -\frac{7}{2}$



$x \in \left(-\infty; -\frac{7}{2}\right)$